Estimating the Consequences of Climate Change from Variation in Weather*

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I formally relate the consequences of climate change to the time series variation in weather extensively explored by recent empirical literature. I show that fixed effects estimators can exactly recover the effects of climate only when agents' payoff functions are in one of two classes. In other cases, fixed effects estimators can only bound the effects of climate, with the direction of the bound determined by how past actions affect the marginal benefit of current actions. Empirical work should begin controlling for forecasts in order to both expand and test the conditions under which it can recover the effects of climate.

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1 Introduction

A pressing empirical agenda seeks to estimate the economic costs of climate change. Ignorance of these costs has severely hampered economists' ability to give concrete policy recommendations. However, while climate primarily varies over space, so too do many unobserved variables that are potentially correlated with climate.[1] Seeking credible identification, an explosively growing empirical literature instead estimates the consequences of transient weather shocks from time series variation in a location's weather.[2] Since climate manifests itself only through weather, the hope is that transient weather shocks identify—or at worst bound—the effects of a change in climate.

Identifying the consequences of climate change from responses to transient weather shocks combines two challenges: (i) empirical researchers must credibly identify the consequences of transient weather shocks, and (ii) the consequences of transient weather shocks must be informative about the consequences of climate change. Challenge (i) is the challenge central to empirical work throughout economics, seeking as-good-as-random assignment of the weather treatment. Empirical researchers have addressed challenge (i) by including time and unit fixed effects, usually taking for granted that the remaining idiosyncratic variation in weather is exogenous.[3] Challenge (ii) is less standard. The recent empirical literature seeks to approximate the effect of one treatment (a change in climate) that is never observed from the estimated effect of a different treatment (a transient change in weather). Whether this mapping between treatments succeeds has been the subject of much discussion but little formal analysis.[4]

I here undertake the first formal analysis that precisely delineates what and how we can learn about the climate from reduced-form estimates of weather impacts. A change in climate differs from a weather shock in being repeated period after period and in affecting expectations of weather far out into the future. Linking weather to climate therefore requires analyzing a dynamic model that captures the distinction between transient and permanent changes in weather. I study an agent (equivalently, firm) who is exposed to stochastic weather outcomes. The agent chooses actions (equivalently, investments) that suit the weather. The

[1] For many years, empirical analyses did rely on cross-sectional variation in climate to identify the economic consequences of climate change (e.g., Mendelsohn et al., 1994; Schlenker et al., 2005; Nordhaus, 2006). However, cross-sectional analyses fell out of favor due to concerns about omitted variables bias. See Dell et al. (2014) and Auffhammer (2018b) for expositions and Massetti and Mendelsohn (2018) for a review.

[2] For recent reviews, see Dell et al. (2014), Carleton and Hsiang (2016), and Heal and Park (2016). Blanc and Schlenker (2017) discuss the strengths and weaknesses of relying on panel variation in weather.

[3] For instance, Dell et al. (2014, 741) write that "the primary advantage of the new literature is identification", and Blanc and Schlenker (2017, 262) describe "weather anomalies" as "ideal right-hand side variables" because "they are random and exogenous". We will see that the existence of forecasts and the likelihood of serial correlation in weather in fact complicate identification.

[4] For instance, Dell et al. (2014, 771–772) emphasize that "short-run changes over annual or other relatively brief periods are not necessarily analogous to the long-run changes in average weather patterns that may occur with climate change."

actions chosen in different periods may be complements or substitutes: when actions are intertemporal complements, choosing a high action in the previous period reduces the cost of choosing a high action today, but when actions are intertemporal substitutes, choosing a high action in the previous period increases the cost of choosing a high action today. The first case is consistent with adjustment costs, and the second case is consistent with actions that require scarce resources, whether physical resources or time.[5] When choosing actions, the agent knows the current weather, has access to forecasts of future weather, and relies on knowledge of the climate to generate forecasts of weather at longer horizons. A change in the climate alters the distribution of potential weather outcomes as well as the agent's expectation of future weather outcomes. I relate both climate change and fixed effects estimators to model primitives and describe their relationship to each other.

I show that researchers can, in general, only bound the effects of climate change on payoffs.[6] Many economists have intuited that short-run adaptation responses to weather are likely to be smaller than long-run adaptation responses to climate (e.g., Deschênes and Greenstone, 2007). I find that two forces can favor this result. First, when actions are durable, a forward-looking agent will undertake more actions in response to a climate shock that also changes the next period's weather than in response to a transient weather shock. Estimating responses to forecasts can capture this channel. Second, when actions are intertemporal complements (as in adjustment cost models), the actions an agent takes in response to a transient weather shock are constrained by the agent's desire to not change actions too much from period to period, but when the same weather shock is repeated period after period, even a myopic agent eventually achieves a larger change in activity through a sequence of incremental adjustments. The latter effect reverses if actions are intertemporal substitutes because agents will then undertake actions in response to transient weather shocks that they would not sustain in the face of a permanent change in climate. In that case, responses to transient weather shocks can overstate responses to permanent changes in climate, a result consistent with conjectures in the literature (e.g., Fisher et al., 2012; Auffhammer and Schlenker, 2014; Blanc and Schlenker, 2017; Auffhammer, 2018b). I show that controlling for lagged actions can indicate whether actions are intertemporal complements or substitutes and thus whether researchers obtain an upper or a lower bound on the effect of climate.

More optimistically, I describe two special cases in which weather regressions can recover

[5] Both types of stories exist in the literature. For instance, in studies of the agricultural impacts of climate change, Deschênes and Greenstone (2007) conjecture that long-run adjustments to changes in climate should be greater than short-run adjustments to weather shocks because there may be costs to adjusting crops, whereas Fisher et al. (2012) and Blanc and Schlenker (2017) emphasize that constraints on storage and groundwater pumping, respectively, could reverse that conclusion.

[6] Much empirical research has sought to estimate the consequences of climate change for profits (e.g., Deschênes and Greenstone, 2007) and for variables such as gross output or income that are potentially related to aggregate payoffs (e.g., Dell et al., 2012; Burke et al., 2015; Deryugina and Hsiang, 2017; Colacito et al., 2019).

the effect of climate. In the first special case, actions are neither intertemporal substitutes nor intertemporal complements: the agent's choice of action in the current period is not directly affected by previous choices. Empirical researchers can then approximate the effects of climate on actions by combining the estimated effects on actions of current weather, lagged weather, and forecasts, which capture the implications of climate change altering present, past, and future weather, respectively.[7] And because empirical researchers can recover the effects of climate on actions, they can also recover the effect of climate on payoffs. However, the standard empirical practice fails to control for forecasts. Fixed effects estimators can then recover the effects of climate in only a narrower range of cases.

A second special case allows empirical researchers to recover the effects of climate from especially simple regressions. This special case holds when the payoff function is a member of a class that includes adjustment cost models, a simple model of resource-dependent costs, a model in which effective adaptation is a constant-returns aggregate of current and past actions, and a model without dynamic linkages. In this case, the marginal effect of climate change does not depend on how actions respond to the climate. The empirical researcher can then recover the effects of climate from a model that ignores forecasts. Further, because forecasts can affect payoffs only through their effects on agents' actions, the researcher can test whether this special case applies by testing whether forecasts affect payoffs.

Figure 1 depicts the intuition for why empirical researchers in general need to recover the effects of climate on actions. Consider estimating the effect of temperature on agricultural profits, as in Deschênes and Greenstone (2007). Each solid curve in the left panel plots profits as a function of current inputs (such as labor and irrigation), conditional on growing season temperature. As we move to the right, the solid curves condition on increasingly warm growing seasons. In static environments, agents maximize profits by choosing inputs at the peaks of these curves, such as points a and b. The dotted line gives the effect on time t profits of time t temperature. Small changes in temperature do not have first-order effects on profits through input choices. This is the content of the envelope theorem.

Thus far, one may be tempted to conclude that we can recover the effect of climate on payoffs by estimating the consequences of transient weather shocks. However, this envelope theorem intuition misses the dynamics that distinguish climate from weather. A change in climate affects past and future weather, not just current weather. First consider the consequences of affecting past weather. Imagine that changing inputs imposes adjustment costs, so that time t profits also depend on time $t-1$ inputs.[8] If last year was hot, then last year's input choices reflect that outcome and it becomes less costly to choose high inputs

[7] Much empirical research has sought to estimate the consequences of climate change for decision variables or functions of decision variables, including productivity (Heal and Park, 2013; Zhang et al., 2018), crime (Ranson, 2014), time allocation (Graff Zivin and Neidell, 2014), and energy use (Auffhammer and Aroonruengsawat, 2011; Deschênes and Greenstone, 2011; Auffhammer, 2018a).

[8] The main text will also explore the implications of resource scarcity stories, which reverse the relationship between time t and time $t-1$ inputs from that described here.

this year. The dashed curve in the left panel of Figure 1 plots profits in a current hot year conditional on having already adjusted last year's input choices in response to last year's being hot. Profits increase at input levels around point b because adjustment costs are reduced. Profits also increase because the optimal input level increases to point c, reflecting that current choices are less constrained by previous choices. Last year's input decisions can therefore have first-order effects on time t profits by changing the adjustment costs faced at time t. Because a transient weather shock will not capture how climate affects the trajectory of previous input decisions, we may expect a transient change in weather to fail to identify the effects of climate.

Now consider the implications of climate affecting future weather. A change in climate leads agents to expect the subsequent year $t + 1$ to once again be hot and thus to expect to choose a high input level in year $t + 1$. Applying more inputs at time t now carries the dynamic benefit of reducing time $t + 1$ adjustment costs. As a result, the dynamically optimal input choice is point d, where the marginal effect on this year's profit is negative but the marginal effect on the present value of expected profits is zero (see equation (1) below).[9] Envelope theorem arguments assume that profit-maximizing inputs always occur where the marginal effect on time t profit is zero. These arguments fail when agents choose inputs with an eye to their implications for future years, whether because current inputs affect future years' adjustment costs, because current inputs affect the availability of resources in future years, or because current inputs are forward-looking adaptation decisions that directly protect against future weather.

To estimate the effect of climate on payoffs, empirical researchers must account for how actions change with the climate. But how can they estimate that change from point a to point d? The right panel of Figure 1 again plots profits as a function of current inputs, but it now holds current weather fixed between curves and instead varies only the previous year's input choices. The curve labeled "ss" depicts profits when the typical temperature has occurred many years in a row, so that previous inputs reached a steady state. The other two curves depict this year's profits under the typical temperature outcome but with higher ("H") and lower ("L") choices of inputs in the previous year. The adjustment costs imposed by these past choices constrain this year's choice of inputs and thereby reduce profits.

The dotted curve gives the effect on myopically optimized profits of changing last year's input choices. For any given previous input choice, the myopically profit-maximizing input choice finds the peak of the curve. The dotted curve has a peak at the myopically optimal labor input implied by curve "ss" because adjustment costs vanish in that case. Around this point (labeled 1), a change in past weather does not have first-order effects through input choices. So the left panel's point c converges to point b. Now imagine that the agent expects the typical temperature to also occur next year. Because this year's input choices

[9] As will be discussed, the dynamically optimal input level could be to the left or the right of point c, but in either case, expecting the subsequent year to be hot would shift the dynamically optimal input level to the right in order to reduce the adjustment costs faced in that subsequent year.

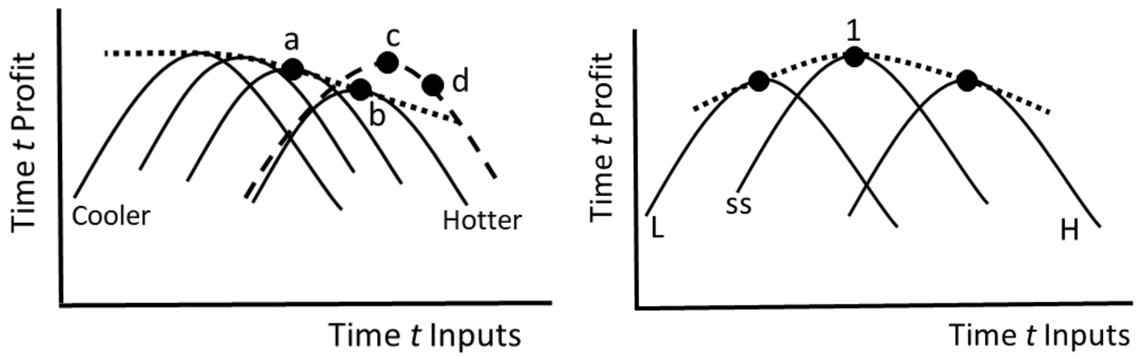


Figure 1: Left: Profits against inputs, conditional on temperature. Temperature is higher for curves farther to the right. The dotted curve through points a and b gives the effect on profits of increasing temperature in the absence of long-run adaptation. Point c accounts for adaptation to previous hot years, and point d accounts for expecting next year to again be hot. Right: Profits against inputs, conditional on past input choices. The curve labeled "ss" sets previous inputs to the steady state that would result if the current temperature had been repeated indefinitely.

do not have first-order effects on next year's profits around point 1, the myopically optimal input choice is also dynamically optimal. So the left panel's point d converges to point c. Combining these results, the left panel's point d converges to point b around point 1, so that the treatment effect of a transient weather shock indeed recovers the effect of permanently changing the weather. The key assumptions are, first, that agents tend to be near their steady-state actions and, second, that small changes in past actions do not have first-order effects on payoffs around a steady state (i.e., that point 1 occurs at a flat point of the dotted line in the right panel). The formal analysis identifies the class of profit functions for which this second assumption holds. Consistent with the intuition given here, it shows that adjustment cost models are indeed members of that class.

Despite the importance of empirically estimating the costs of climate change and the sharpness of informal debates around the relevance of the recent empirical literature to climate change, there has been remarkably little formal analysis of the economic link between weather and climate. The primary exception is an argument given in Hsiang (2016) and again in Deryugina and Hsiang (2017). The argument stipulates that a change in climate differs from a change in weather only by affecting beliefs about future weather. This difference in beliefs can matter for payoffs only if it affects an agent's chosen actions. However, the envelope theorem tells us that an optimizing agent's actions cannot have first-order consequences for payoffs. Therefore the effects of weather on payoffs exactly—and generically—identify the effects of climate on payoffs.

By formalizing the distinction between climate and weather in a dynamic environment, the present analysis highlights two weak points in this argument. First, it is true that a change in climate alters beliefs about future weather, but it is also true that a change in climate alters past weather and thus past actions. Past actions are predetermined variables from the perspective of an optimizing agent and thus do not drop out through the envelope theorem. Illustrating that beliefs are not the only difference between weather and climate, we will see that even myopic agents can respond differently to weather and climate. Second, in a dynamic model, the envelope theorem applies to the intertemporal value function, not to the per-period payoff function investigated by much empirical work. In a dynamic setting, optimized current actions can have first-order effects on current payoffs when those are offset by first-order effects on expected future payoffs. The present analysis describes the special class of payoff functions for which optimized current actions do not have first-order effects on future payoffs around a steady state and thus, from the first-order condition, also do not have first-order effects on current payoffs around a steady state. In this case, the effects of current actions around a steady state comport with the envelope theorem's implications in a static environment.[10]

A few other lines of research are also related. First, calibrated numerical simulations have shown that dynamic responses are critical to the effects of climate on timber markets (Sohngen and Mendelsohn, 1998; Guo and Costello, 2013) and to the cost of increased cyclone risk (Bakkensen and Barrage, 2018). I develop a general analytic setting that precisely disentangles several types of dynamic responses and relates them to widely used fixed effects estimators. Second, some empirical papers have demonstrated that actions do respond to forecasts of future weather (e.g., Neidell, 2009; Rosenzweig and Udry, 2013, 2014; Wood et al., 2014; Miller, 2015).[11] In particular, Shrader (2017) and Taraz (2017) use variation in forecasts and past weather outcomes, respectively, to estimate ex-ante adaptation to weather events. I formally demonstrate that it is critical to estimate responses both to forecasts and to lagged weather when seeking to learn about the consequences of climate change. Finally, Kelly et al. (2005) and Kala (2017) study learning about the climate from observed weather. I here abstract from learning in order to focus on mechanisms more relevant to the recent empirical literature.[12]

[10]In an earlier expositional analysis, I showed how envelope theorem arguments can fail in a three-period model (Lemoine, 2017). The present work precisely analyzes the consequences of climate change in an infinite-horizon model and constructively shows which types of empirical estimates can be informative about the climate.

[11]Severen et al. (2018) show that land markets capitalize expectations of future climate change and correct cross-sectional analyses in the tradition of Mendelsohn et al. (1994) for this effect. In contrast, I here study responses to widely available, shorter-run forecasts in a time series context and show how to use them to improve panel analyses in the tradition of Deschênes and Greenstone (2007).

[12]Kelly et al. (2005) frame the cost of learning as an adjustment cost. Quiggin and Horowitz (1999, 2003) discuss broader costs of adjusting to a change in climate. These papers' adjustment costs are conceptually distinct from the adjustment costs studied here. I follow the empirical literature in studying the long-run cost of changing the climate without modeling the transition from one climate to another (Carleton et al.,

The challenge of attempting to estimate long-run effects from short-run variation is a common one in empirical economics. I give three examples. In environmental economics, researchers would like to estimate the long-run health consequences of pollution but typically only have exogenous variation in short-run exposure to pollution. Some recent work has found exogenous, policy-induced variation in long-run pollution exposure (e.g., Chen et al., 2013; Anderson, 2015; Barreca et al., 2017). This type of variation is unlikely to be available to researchers interested in the consequences of changing the climate.

In labor economics, a large reduced-form literature investigates the consequences for employment of increasing the minimum wage. Sorkin (2015) argues that this literature has been estimating only short-run effects: firms will not reduce employment by much when they anticipate that inflation will eat away observed increases in the minimum wage and face adjustment costs in changing the number of jobs they offer. He shows that reduced-form regressions can recover long-run consequences only in special cases, which include the requirement that employment initially be near a steady state. A similar requirement will be an element of the special cases in which weather variation can identify the effects of climate.[13]

Finally, macroeconomists study the tradeoff between output and inflation. Economists had hoped to learn about long-run tradeoffs by estimating distributed lag models, but Lucas (1972) argued that, when agents have rational expectations, the lagged response to a transient inflation shock is not informative about the long-run effects of permanently changing inflation policy. In the present setting, a change in climate is analogous to shifting the "policy rule" governing weather. I will show that differences arise both from rational expectations and from having reacted to alterations in past weather.

The next section describes the setting. Section 3 analyzes the effect of climate on payoffs, including how to estimate these effects in the special case in which effects on actions vanish. Section 4 considers estimating the effect of climate on actions and Section 5 builds on these results to estimate the effect of climate on payoffs in general cases. Section 6 examines the effects of aggregating over longer timesteps, including in recent long difference estimators. The final section summarizes caveats and potential extensions. The appendix contains proofs.

2018, is a notable exception). The present use of "adjustment costs" follows much other economics literature in referring to the cost of changing decisions from their previous levels, where decisions here respond to variations in weather. The present paper studies how these adjustment costs hinder estimation of the consequences of climate change from weather, not how they affect the cost of transitioning from one climate to another. I return to this point in the conclusion.

[13] Hamermesh (1995) argues that the pre-period before a minimum wage increase is not actually a pre-period because firms have foreknowledge of the change and may use that knowledge to begin adjusting, and he argues that the post-period may not capture long-run effects unless employers can adjust quickly. In the present setting, similar considerations will complicate identification of climate effects from weather variation and will require researchers to estimate responses to forecasts.

2 Setting

An agent is repeatedly exposed to stochastic weather outcomes and takes actions based on realized weather and information about future weather. The realized weather in period t is w_t and the agent's chosen action is A_t.[14] This action may be interpreted as a level of activity (e.g., time spent outdoors, energy used for heating or cooling, irrigation applied to a field) or as a stock of capital (e.g., outdoor gear, size or efficiency of furnace, number or efficiency of irrigation lines). The agent's time t payoffs are $\pi(A_t, A_{t-1}, w_t, w_{t-1})$, which is twice-differentiable. Letting subscripts indicate partial derivatives with respect to the indicated argument, I assume declining marginal benefits of current and past actions ($\pi_{11} < 0, \pi_{22} \leq 0$).

I interpret actions as adaptations that become more valuable with high weather outcomes ($\pi_{13}, \pi_{23} \geq 0$). Following terminology from the literature on climate adaptation (e.g., Fankhauser et al., 1999; Mendelsohn, 2000), a case with $\pi_{13} > 0$ reflects adaptation that can occur after weather is realized ("reactive" or "ex-post" adaptation) and a case with $\pi_{23} > 0$ reflects adaptation that can occur before weather is realized ("anticipatory" or "ex-ante" adaptation).[15] I allow for adaptation to play both roles at once. The possibility that $\pi_4 \neq 0$ reflects potential delayed impacts from the previous period's weather, with π_{14} and π_{24} capturing the potential for ex-post adaptation to alter these delayed impacts. Consistent with the normalizations above, I assume $\pi_{14}, \pi_{24} \geq 0$

I allow π_{12} to be positive or negative, with its magnitude constrained as described below. When $\pi_{12} < 0$, actions are "intertemporal substitutes", so that choosing a higher level of past actions increases the cost of choosing higher actions today. I describe this case as a resource scarcity story.[16] For instance, pumping groundwater today raises the cost of pumping groundwater tomorrow, or calling in sick today increases the cost of calling in sick tomorrow. When $\pi_{12} > 0$, actions are "intertemporal complements", so that choosing a higher level of past actions increases the benefit from choosing higher actions today. I describe this case as an adjustment cost story.[17,18] For instance, small changes to cropping practices or work schedules may be easier to implement than large changes. The magnitude of π_{12} affects the agent's preferred timing of adaptation. As $|\pi_{12}|$ becomes large, the agent

[14] For expositional purposes, I treat actions and the weather index as being one-dimensional. Generalizing to vector-valued actions and weather is straightforward but increases notation without further insight.

[15] If we interpret actions as the choice of capital stock, then a model with only ex-ante adaptation corresponds to a time-to-build model with a one-period lag and full depreciation.

[16] Relating to the literature on resource extraction, the case with $\pi_{12} < 0$ can be seen as reflecting stock-dependent extraction costs (Heal, 1976).

[17] The benchmark quadratic adjustment cost model has $\pi_{12} = k$ for some $k > 0$ (see Hamermesh and Pfann, 1996). If we interpret actions as the choice of capital stock, then quadratic adjustment costs correspond to models with quadratic investment costs. Some evidence suggests that the costs of adjusting capital and labor are small in practice (e.g., Hall, 2004), though others report convex adjustment costs for labor (e.g., Blatter et al., 2012) and capital (e.g., Cooper and Haltiwanger, 2006).

[18] This case also corresponds to shifts in preferences due to habit formation, as studied by Ge and Ho (2019) in the context of heating and cooling decisions.

prefers to begin adapting before the weather event arrives, but when $|\pi_{12}|$ is small, the agent may wait to undertake most adaptation only once the weather event has arrived.[19]

The agent observes time t weather before selecting her time t action. The agent also understands the climate C, which controls the distribution of weather. We can interpret weather as temperature and climate as a location's long-run average temperature. At all times before $t-1$, the agent's only information about time t weather consists in knowledge of the climate. However, at time $t-1$ the agent receives a forecast f_{t-1} of time t weather: $f_{t-1} = C + \zeta\nu_{t-1}$, where the innovation ν_{t-1} is a mean-zero, serially uncorrelated random variable with variance $\tau^2 > 0$. The forecast is an unbiased predictor of time t weather: $w_t = f_{t-1} + \zeta\epsilon_t$, where ϵ_t is a mean-zero, serially uncorrelated random variable with variance $\sigma^2 > 0$.[20] The parameter $\zeta \geq 0$ is a perturbation parameter that will be useful for analysis (see Judd, 1996). The covariance between ϵ_t and ν_t is ρ, which implies that the covariance between w_t and w_{t-1} is $\zeta^2\rho$.

The agent maximizes the present value of payoffs over an infinite horizon:

$$\max_{\{A_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_0 \left[\pi(A_t, A_{t-1}, w_t, w_{t-1})\right],$$

where $\beta \in [0,1)$ is the per-period discount factor, A_{-1} is given, and E_0 denotes expectations at the time 0 information set. The solution satisfies the following Bellman equation:

$$\begin{aligned} V(A_{t-1}, w_t, f_t, w_{t-1}; \zeta) = &\max_{A_t} \left\{ \pi(A_t, A_{t-1}, w_t, w_{t-1}) + \beta E_t \left[V(A_t, w_{t+1}, f_{t+1}, w_t; \zeta)\right] \right\} \\ \text{s.t.} \quad w_{t+1} = &f_t + \zeta\epsilon_{t+1} \\ f_{t+1} = &C + \zeta\nu_{t+1}. \end{aligned}$$

The setting is sufficiently general to describe many applications of interest. For instance, much empirical literature has studied the effects of weather on energy use. The agent could then be choosing indoor temperature in each period, where payoffs depend on current actions through energy use and depend on weather through thermal comfort. Much empirical work

[19] The magnitude of π_{12} is related to the distinction between ex-post and ex-ante adaptation insofar as it affects the agent's preferred timing of adaptation actions. However, π_{12} incentivizes early adaptation only to reduce the costs of later adaptation, not because early adaptation provides protection from weather events. I reserve the terms ex-ante and ex-post adaptation to refer to the effects of actions on the marginal benefit of weather, captured by π_{13}, π_{23}, π_{14}, and π_{24}.

[20] Consistent with much previous literature, climate here controls average weather. One might wonder about the dependence of higher moments of the weather distribution on climate. In fact, the effects of climate change on the variance of the weather are poorly understood and likely to be spatially heterogeneous (e.g., Huntingford et al., 2013; Lemoine and Kapnick, 2016). Further, for economic analysis, we need to know not just how climate change affects the variance of realized weather but how it affects the forecastability of weather: the variance of the weather more than one period ahead is $\zeta^2(\sigma^2 + \tau^2)$, so we need to apportion any change in variance between σ^2 and τ^2. I leave such an extension to future work.

has also studied the effect of weather on labor productivity. The decision variable could be effort, the dependence of payoffs on weather can reflect current thermal stress as well as the effects of the previous day's weather via sleep and physiological functioning, the resource scarcity is one of tasks needing to be done, and forecasts allow the agent to plan tasks and vacation time around weather outcomes.

Whereas forecasts have clear interpretations in shorter-run decisions about energy use and time allocation, one may wonder about the existence of forecasts over longer timesteps, such as when studying the effect of growing season degree days on agricultural profits.[21] Much work has shown that seasonal forecasts can be valuable and useful to developing country farmers with rainfed agriculture (e.g., Hansen et al., 2011). Indian farmers do adjust planting stage decisions to seasonal monsoon forecasts (Rosenzweig and Udry, 2013; Miller, 2015), and migration decisions also respond to these forecasts (Rosenzweig and Udry, 2014).[22] Further, Indian farmers develop more accurate beliefs about the start of the monsoon when such accuracy would be especially valuable (Giné et al., 2015). In a developed country context, Takle et al. (2013) describe the various seasonal forecasts of interest to U.S. corn farmers. In these contexts, the decision variables can involve labor, fertilizer, irrigation, or crop varieties, and weather costs can reflect the losses in crop yields.[23]

I will often impose one of the following two assumptions:

Assumption 1. *ζ^2 is small.*

Assumption 2. *π is quadratic.*

Either assumption will limit the consequences of stochasticity for optimal policy, whether by limiting the variance of weather outcomes (Assumption 1) or by making the policy function independent of that variance (Assumption 2).[24]

I will be interested in estimating the effects of changing C from responses to time series variation in w_t and f_t. It is important to be clear about the climate experiment. I study the effects of a change in climate on an agent who has had time to adapt to the new climate. This climate change treatment is consistent with the exercise common in the empirical literature, which calculates the effect of changing today's distribution of weather to a distribution projected to hold by the end of the century. I will not study how the transition from

[21]Based on day-to-day experience, many believe that forecasts are useless past a week or two, but in fact forecast skill starts increasing at longer horizons as ocean-atmosphere interactions dominate uncertainty about initial conditions. See National Research Council (1999), National Academies of Sciences (2016), and Klemm and McPherson (2017), among others, for reviews of seasonal forecasting, including its use in agriculture.

[22]In a different context, Shrader (2017) shows that U.S. fishers respond to seasonal El Niño forecasts.

[23]Scott (2013) estimates how the returns to planting a field with crops depend on the previous state of the field, which maps into the present setting's π_{12}.

[24]When applying Assumption 2, the chosen policy is still affected by the variance of weather because that chosen policy depends on the realized weather.

one climate to another interacts with agents' decisions, whether with the decisions already observable in empirical data or with the decisions that will be influenced by future climate change. I will also not study how expectations of a future change in climate affect agents today. These are all important questions but beyond the scope of the present analysis and largely outside of the reduced-form empirical literature that this analysis seeks to inform.[25] The conclusion returns to some of these points.

3 The Effect of Climate on Payoffs

Empirical researchers want to credibly identify the effects of climate change on the payoffs of households and firms. The ideal dataset would contain panel variation in a forward-looking measure of value, such as land values or other asset prices, that could be combined with panel variation in expectations of future climate to estimate the expected impacts of climate change. However, researchers instead have panel variation in flow payoffs and panel variation in weather. For instance, much empirical research studies how variation in weather affects agricultural profits (e.g., Deschênes and Greenstone, 2007) and variables such as gross output or income that are potentially related to aggregate payoffs (e.g., Dell et al., 2012; Burke et al., 2015; Deryugina and Hsiang, 2017; Colacito et al., 2019). I now consider how to estimate the effect of climate on long-run payoffs from time series variation in weather and flow payoffs.

Consider a deterministic system, with $\zeta = 0$. The first-order condition is:

$$0 = \pi_1(A_t, A_{t-1}, C, C) + \beta V_1(A_t, C, C, C; 0).$$

The envelope theorem yields:

$$V_1(A_{t-1}, C, C, C; 0) = \pi_2(A_t, A_{t-1}, C, C).$$

Advancing this forward by one timestep and substituting into the first-order condition, we have the Euler equation:

$$0 = \pi_1(A_t, A_{t-1}, C, C) + \beta \pi_2(A_{t+1}, A_t, C, C). \tag{1}$$

If $\pi_2 = 0$, then we have a static optimization problem and the agent maximizes payoffs by setting the marginal time t benefit of time t actions to zero. In Figure 1, points a, b, and c would in fact be optimal. However, matters are different if π_2 is nonzero. A case with $\pi_2 > 0$ is a case of deferred benefits: actions taken in period t provide benefits in the future, as with capital investments. In the presence of these benefits, the optimal time t action must have a negative marginal flow benefit in time t, as with point d in Figure 1. A case with $\pi_2 < 0$

[25] As an exception, Carleton et al. (2018) study the implications of climate change as it may unfold over the coming century.

is a case of deferred costs: actions taken in period t impose costs in the future, as with the use of scarce resources or as when taking out a loan. In the presence of deferred costs, the optimal time t action must have a positive marginal flow benefit in time t.[26]

A steady state $\bar{A}$ of the deterministic system is implicitly defined by

$$0 = \pi_1(\bar{A}, \bar{A}, C, C) + \beta\pi_2(\bar{A}, \bar{A}, C, C). \tag{2}$$

Define $\bar{\pi} \triangleq \pi(\bar{A}, \bar{A}, C, C)$. The following lemma describes the uniqueness and stability of the steady state.

Lemma 1. *$\bar{A}$ is locally saddle-path stable if and only if $(1+\beta)|\bar{\pi}_{12}| < -\bar{\pi}_{11} - \beta\bar{\pi}_{22}$, in which case $\bar{A}$ is unique.*

Proof. See appendix. □

I henceforth assume that $(1+\beta)|\bar{\pi}_{12}| < -\bar{\pi}_{11} - \beta\bar{\pi}_{22}$, so that the deterministic steady state is unique and saddle-path stable.

Now consider optimal policy in the stochastic system. The first-order condition is:

$$0 = \pi_1(A_t, A_{t-1}, w_t, w_{t-1}) + \beta E_t[V_1(A_t, w_{t+1}, f_{t+1}, w_t; \zeta)].$$

The envelope theorem yields:

$$V_1(A_{t-1}, w_t, f_t, w_{t-1}; \zeta) = \pi_2(A_t, A_{t-1}, w_t, w_{t-1}).$$

Advancing this forward by one timestep and substituting into the first-order condition, we have the stochastic Euler equation:

$$0 = \pi_1(A_t, A_{t-1}, w_t, w_{t-1}) + \beta E_t[\pi_2(A_{t+1}, A_t, w_{t+1}, w_t)].$$

I analyze the stochastic system by approximating around the steady state and $\zeta = 0$ (Judd, 1996).

Using either Assumption 1 or Assumption 2, we have:

$$\begin{aligned} E_0[\pi(A_t, A_{t-1}, w_t, w_{t-1})] =& \bar{\pi} + \bar{\pi}_1(E_0[A_t] - \bar{A}) + \bar{\pi}_2(E_0[A_{t-1}] - \bar{A}) \\ &+ \frac{1}{2}\bar{\pi}_{11}E_0[(A_t - \bar{A})^2] + \frac{1}{2}\bar{\pi}_{22}E_0[(A_{t-1} - \bar{A})^2] + \frac{1}{2}(\bar{\pi}_{33} + \bar{\pi}_{44})\zeta^2(\sigma^2 + \tau^2) \\ &+ \bar{\pi}_{12}E_0[(A_t - \bar{A})(A_{t-1} - \bar{A})] + \bar{\pi}_{13}Cov_0[A_t, w_t] + \bar{\pi}_{23}Cov_0[A_{t-1}, w_t] \\ &+ \bar{\pi}_{14}Cov_0[A_t, w_{t-1}] + \bar{\pi}_{24}Cov_0[A_{t-1}, w_{t-1}] + \bar{\pi}_{34}\zeta^2\rho, \end{aligned} \tag{3}$$

for $t > 1$. The following lemma describes the evolution of $E_0[A_t]$.

[26] In Figure 1, the dynamically optimal actions would now be to the left of points a, b, and c. The effect of expecting the subsequent year to be hot would still shift that action to the right, so point d would still be to the right of the corrected point c.

Lemma 2. *Let either Assumption 1 or 2 hold, and let $E_0[(A_1 - \bar{A})^2]$ be small. Then, as $t \to \infty$, $E_0[A_t] \to \bar{A}$.*

Proof. See appendix. □

Using this result, differentiating equation (3) with respect to C, and applying either Assumption 1 or Assumption 2 again, we find that, as t becomes large,

$$\frac{\mathrm{d}E_0[\pi(A_t, A_{t-1}, w_t, w_{t-1})]}{\mathrm{d}C} \to \bar{\pi}_3 + \bar{\pi}_4 + [\bar{\pi}_1 + \bar{\pi}_2]\frac{\mathrm{d}\bar{A}}{\mathrm{d}C}. \tag{4}$$

The marginal effect of climate on long-run payoffs is composed of the direct effect of a larger weather index, in both the present ($\bar{\pi}_3$) and the past ($\bar{\pi}_4$), and the effects of changing long-run actions, including both present actions ($\bar{\pi}_1$) and past actions ($\bar{\pi}_2$). From the Euler equation (1), we have $\bar{\pi}_1 = -\beta\bar{\pi}_2$, so as t becomes large,

$$\frac{\mathrm{d}E_0[\pi(A_t, A_{t-1}, w_t, w_{t-1})]}{\mathrm{d}C} \to \bar{\pi}_3 + \bar{\pi}_4 + (1-\beta)\bar{\pi}_2\frac{\mathrm{d}\bar{A}}{\mathrm{d}C}. \tag{5}$$

Whether economic responses increase or decrease payoffs depends on the sign of $\bar{\pi}_2$. As described in Section 4, a case with $\bar{\pi}_2 > 0$ is a case in which higher actions impose costs today but provide benefits tomorrow, as when undertaking adaptation investments that take time to build. A case with $\bar{\pi}_2 < 0$ is a case in which higher actions provide benefits today but impose costs tomorrow, as when borrowing money or selling from storage. Undertaking more actions because of climate change increases payoffs if and only if actions are of the former type.

I now analyze the implications of a special class of payoff functions for our ability to estimate the effect of climate on payoffs from within-unit weather variation. For these payoff functions, the marginal benefit of past actions around a steady state is proportional to the marginal benefit of current actions around a steady state:

Assumption 3. $\pi_2(A_t, A_{t-1}, w_t, w_{t-1}) = K\pi_1(A_t, A_{t-1}, w_t, w_{t-1})$ *if* $A_{t-1} = A_t$*, for* $K \neq -\beta$.

Consider a few members of this class. First, adjustment cost models yield $K = 0$: if $\pi = g(A_t, (A_t - A_{t-1})^z, w_t, w_{t-1})$ for $z > 1$, then $\pi_2 = z(A_t - A_{t-1})^{z-1} g_2(A_t, (A_t - A_{t-1})^z, w_t, w_{t-1})$ and thus is equal to 0 when $A_t = A_{t-1}$. Second, a model in which the returns to resource extraction decline in previous extraction can yield $K = -1$: if $\pi = g(A_t/A_{t-1}, w_t, w_{t-1})$, then $\pi_1 = g_1(A_t/A_{t-1}, w_t, w_{t-1})/A_{t-1}$ and $\pi_2 = -A_t g_1(A_t/A_{t-1}, w_t, w_{t-1})/(A_{t-1}^2)$. Third, a model in which ex post adaptation and ex ante adaptation actions form a constant elasticity of substitution (CES) aggregate with distribution parameter κ yields $K = (1-\kappa)/\kappa$: $\pi = g(h(A_t, A_{t-1}), w_t, w_{t-1})$ where $h(A_t, A_{t-1}) = (\kappa A_t^\sigma + (1-\kappa)A_{t-1}^\sigma)^{1/\sigma}$ for $\sigma < 1, \neq 0$ and $h(A_t, A_{t-1}) \to A_t^\kappa A_{t-1}^{1-\kappa}$ as $\sigma \to 0$. Finally, a model without dynamic linkages has $\pi_2(\cdot,\cdot,\cdot,\cdot) = 0$ and thus $K = 0$.

Empirical researchers hope to recover (5) from time series variation in weather. Label agents by j and imagine that they are in the same climate C with the same payoff function π and the same stochastic process driving forecasts and weather, though each agent may draw a different sequence of weather and forecasts. Most empirical researchers will not observe the full set of actions available to agents or firms. As a result, empirical researchers may estimate the following regression:

$$\pi_{jt} = \alpha_j + \psi_t + X_{jt}\theta + \eta_{jt}, \tag{6}$$

where α_j is a fixed effect for agent j, ψ_t is a time fixed effect, and η_{jt} is an error term.[27] The vector of covariates X_{jt} is

$$\begin{bmatrix} w_{jt} & f_{jt} & w_{j(t-1)} & f_{j(t-1)} & \dots & w_{j(t-K)} & f_{j(t-K)} \end{bmatrix}.$$

We are interested in the vector of coefficients θ. I denote each element with a subscript corresponding to the covariate it multiplies, and I use a hat to denote the probability limit of each element.

Proposition 1. *Let Assumption 3 hold. Further, let either Assumption 1 hold or let Assumption 2 hold with the ϵ and ν normally distributed. And let $(A_{j(t-1)} - \bar{A})^2$ and $(A_{jt} - \bar{A})^2$ be small for all observations and let each agent's average actions be $\bar{A}$. Then, for s large and $K > 0$,*

$$\mathrm{d}E_0[\pi_s]/\,\mathrm{d}C = \bar{\pi}_3 + \bar{\pi}_4 = \hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}}$$

and

$$\hat{\theta}_{f_t} = 0.$$

Proof. See appendix. □

From equation (4), estimating the effects of climate on payoffs requires estimating four terms: $\bar{\pi}_3$, $\bar{\pi}_4$, $\bar{\pi}_1[\mathrm{d}\bar{A}/\,\mathrm{d}C]$, and $\bar{\pi}_2[\mathrm{d}\bar{A}/\,\mathrm{d}C]$. However, the proposition shows that when Assumption 3 holds, the direct effects on weather suffice to describe the effect of climate on payoffs. Assumption 3 and the Euler equation (1) imply that $\bar{\pi}_2 = \bar{\pi}_1 = 0$: an optimizing agent sets the marginal benefit of actions to zero around a steady state, as with point 1 in Figure 1. In this case, the consequences of marginal climate change are independent of

[27] I do not explicitly model the unobservable characteristics that motivate the fixed effects specification because they are not central to the question of interest. These unobservables relate to challenge (i) described in the introduction. See Dell et al. (2014) and Auffhammer (2018b), among others, for standard expositions of identification in the climate-economy literature. I assume that the only sources of omitted variables bias are the failure to control for variables, such as forecasts or lagged actions, that are defined within the theoretical model.

changes in actions.[28] Therefore, summing $\hat{\theta}_{w_t}$ and $\hat{\theta}_{w_{t-1}}$ fully captures the effects of climate on payoffs. Further, we can test whether Assumption 3 holds by examining the magnitude of the coefficient on forecasts: because forecasts matter for current payoffs only through their effects on actions, they cannot affect these payoffs if Assumption 3 indeed holds and agents are near a steady state.[29]

However, Assumption 3 will not hold in general. We cannot then ignore the effects of climate on agents' actions when estimating effects on climate. I next study how to recover the effect of climate on actions before returning to estimation of the effect of climate on payoffs.

4 Estimating the Effect of Climate on Actions

We have seen that the effects of climate on payoffs are closely related to its effects on actions. Further, much empirical research has sought to estimate the consequences of climate change for decision variables or functions of decision variables, including productivity (Heal and Park, 2013; Zhang et al., 2018), time allocation (Graff Zivin and Neidell, 2014), and energy use (Auffhammer and Aroonruengsawat, 2011; Deschênes and Greenstone, 2011; Auffhammer, 2018a). We therefore now consider how to estimate the effect of climate on actions from time series variation in weather.

The proof of Lemma 2 shows that if either Assumption 1 or 2 holds and $(A_{t-1} - \bar{A})^2$ is small, then

$$\begin{aligned} A_t =& \bar{A} + \overbrace{\frac{\bar{\pi}_{14}}{\chi}(w_{t-1} - C) + \frac{\bar{\pi}_{12}}{\chi}(A_{t-1} - \bar{A})}^{\text{effects of past weather}} + \overbrace{\frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}}{\chi}(w_t - C)}^{\text{effects of current weather}} \\ &+ \underbrace{\frac{\beta\bar{\pi}_{23} + \beta(\bar{\pi}_{13} + \beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}}{\chi}(f_t - C)}_{\text{effects of future weather}}, \end{aligned} \tag{7}$$

[28] As described earlier, one of the special cases of Assumption 3 is a model with no dynamic linkages ($\pi_2(\cdot, \cdot, \cdot, \cdot) = 0$), in which case the agent solves a series of independent, static decision problems. We have therefore recovered the result obtained by previous appeals to the envelope theorem in static settings (Hsiang, 2016; Deryugina and Hsiang, 2017), but we now see that those settings were a rather special case: envelope theorem arguments do not suffice to make actions irrelevant in a model with dynamic linkages. See also footnote 36.

[29] Much literature has studied dependent variables such as crop yields (e.g., Schlenker and Roberts, 2009), mortality (e.g., Deschênes and Moretti, 2009; Deschênes and Greenstone, 2011), and health (e.g., Deschenes, 2014) that are functions of actions but are not payoff functions. If we consider recovering the effects of climate on such dependent variables from a fixed effects regression on weather, then the analogue of Proposition 1 no longer applies because the Euler equation (1) holds only for payoffs, not for other functions of actions. However, the results of Section 5 will extend to these types of dependent variables.

where $\chi > |\pi_{12}|$. We see time t actions determined by past, current, and future weather. Actions depend on past weather in two ways. First, past weather affects the marginal payoffs from current actions directly when $\bar{\pi}_{14} \neq 0$. This is a form of ex-post adaptation. Second, past weather affects past actions, and these past actions affect current actions when $\bar{\pi}_{12} \neq 0$.[30] When actions are intertemporal complements ($\bar{\pi}_{12} > 0$), high values of past actions justify higher actions today, but when actions are intertemporal substitutes ($\bar{\pi}_{12} < 0$), high values of past actions justify lower actions today. In the former case, maintaining the high action over time reduces adjustment costs, but in the latter case, the past high action depletes the resources needed to maintain a high action today.

Actions also depend on current weather, in three ways. First, actions respond to current weather as a means of mitigating its immediate harm or amplifying its immediate benefits. This channel is controlled by $\bar{\pi}_{13}$. Second, actions respond to current weather when current actions can mitigate the harm or amplify the benefits incurred by current weather in future periods. This channel is controlled by $\bar{\pi}_{24}$ and arises only for forward-looking agents. As an example of the distinction between the two channels, an agent may avoid going outside on a cold day both to minimize discomfort from the current temperature and to avoid getting sick in the near future. Both of these channels are forms of ex-post adaptation. Third, when $\bar{\pi}_{14} \neq 0$, the current weather will affect the agent's chosen action in the next period. A forward-looking agent adjusts her current action in preparation for that choice. This channel vanishes when $\bar{\pi}_{12} = 0$ because today's actions then do not directly interact with subsequent actions.

Finally, actions also depend on future weather, both directly and indirectly. The direct channel reflects the possibility of ex-ante adaptation, controlled by $\bar{\pi}_{23}$. When today's actions are durable investments that can control the effects of future weather, the agent chooses today's actions based on expectations of that future weather. The indirect channel reflects how the agent begins adjusting actions today in anticipation of the actions she will want to take in the subsequent period. When the agent receives a higher forecast, she expects to take a higher action in the subsequent period, controlled by $\bar{\pi}_{13}$ and $\beta\bar{\pi}_{24}$. When $\bar{\pi}_{12} > 0$, the indirect channel leads the agent to choose high actions today as a means of reducing adjustment costs, but when $\bar{\pi}_{12} < 0$, the indirect channel leads the agent to choose low actions today as a means of conserving resources.

We want to know how long-run actions change, on average, with the climate index C.

Lemma 3. *Let either Assumption 1 or 2 hold, and let $E_0[(A_1 - \bar{A})^2]$ be small. Then, as $t \to \infty$,*

$$\frac{\mathrm{d}E_0[A_t]}{\mathrm{d}C} \to \frac{\mathrm{d}\bar{A}}{\mathrm{d}C} = \frac{\overbrace{\bar{\pi}_{13} + \bar{\pi}_{14} + \beta\bar{\pi}_{24}}^{\textit{ex-post}} + \overbrace{\beta\bar{\pi}_{23}}^{\textit{ex-ante}}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \geq 0. \tag{8}$$

[30] Because past actions are also affected by expectations of current weather, it is more precise to say that current actions depend on past weather and past forecasts, not just past weather.

Proof. The given conditions imply that Lemma 2 holds. The result follows from differentiating equation (2) with respect to C via the implicit function theorem. □

Expected future actions increase in the climate index because I normalized high actions to be more beneficial when the weather index is large. Equation (8) captures how climate change alters weather in all periods: the past, the present, and the future. We see the various forms of ex-post adaptation captured by $\bar{\pi}_{13}$, $\bar{\pi}_{14}$, and $\beta\bar{\pi}_{24}$. We also see the possibility of ex-ante adaptation, controlled by $\bar{\pi}_{23}$ and arising because the agent understands that the altered climate affects weather in subsequent periods. Finally, observe that $\bar{\pi}_{12}$ enters through the denominator in (8). When actions are intertemporal substitutes ($\bar{\pi}_{12} < 0$), this term reduces the magnitude of the response to climate change, as when resource scarcity make long-run responses smaller than short-run responses. However, when actions are intertemporal complements ($\bar{\pi}_{12} > 0$), this term increases the magnitude of the response to climate change, as when adjustment costs allow long-run responses to exceed short-run responses.

Empirical researchers hope to recover (8) from time series variation in weather. Consider the following fixed effects regression:

$$A_{jt} = \alpha_j + \psi_t + \Gamma_1 w_{jt} + \Gamma_2 w_{j(t-1)} + \Gamma_3 f_{jt} + \Gamma_4 A_{j(t-1)} + \eta_{jt}, \tag{9}$$

where I again label firms by j and η_{jt} is an error term that is uncorrelated with the covariates. I again use a hat to denote the probability limit of each estimator. The following lemma relates the estimated coefficients to the effect of climate change.

Lemma 4. *Let either Assumption 1 or 2 hold, and let $(A_{j(t-1)} - \bar{A})^2$ be small for all observations. Then:*

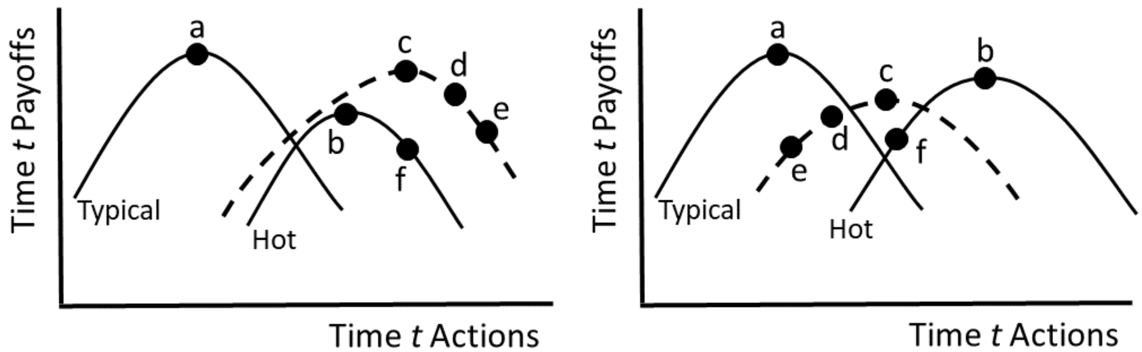
$$\hat{\Gamma}_1 + \hat{\Gamma}_2 + \hat{\Gamma}_3 = \omega \left(\frac{\mathrm{d}\bar{A}}{\mathrm{d}C} + \beta \frac{\bar{\pi}_{14} + \bar{\pi}_{13} + \beta\bar{\pi}_{24}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \Omega \right), \tag{10}$$

where $\bar{\pi}_{12} > 0$ implies $\omega \in (0, 1)$ and $\Omega > 0$ and $\bar{\pi}_{12} < 0$ implies $\omega > 1$ and $\Omega < 0$.

Proof. See appendix. □

The three coefficients capture the three temporal relationships altered by climate change: $\hat{\Gamma}_1$ recovers consequences of altering current weather, $\hat{\Gamma}_2$ recovers consequences of altering past weather, and $\hat{\Gamma}_3$ recovers consequences of altering expectations of future weather. However, we cannot in general recover the response to a permanent change in climate from the estimated response to transient weather shocks. The reason for this failure is the possibility that $\bar{\pi}_{12} \neq 0$.

Relationships of intertemporal substitutability or complementarity drive two types of wedges between the estimator in (10) and the effect of climate change in (8). First consider adjustment cost stories, which have $\bar{\pi}_{12} > 0$. The second term in parentheses in (10) reflects how the agent adjusts today's actions in expectation of today's weather and forecast changing



(a) Adjustment costs (b) Resource scarcity

Figure 2: The effects of climate and of transient weather shocks on actions, for cases with adjustment costs (left) and resource scarcity (right). Points a, b, c, and d are as in Figure 1. Point e depicts a case in which expected changes in weather are transient, and point f depicts a case in which previous changes in weather were only transient.

tomorrow's desired actions. Fixed effects estimators reflect idiosyncratic variation in weather, so they reflect an agent desiring to choose different actions over time. In the presence of adjustment costs, the agent shifts today's actions towards the level that will be chosen tomorrow ($\Omega > 0$). This effect vanishes as actions become more similar between today and tomorrow, so it tends to make today's actions more responsive to idiosyncratic weather shocks than to changes in climate that alter weather in all periods. The $\omega \in (0, 1)$ captures how adjustment costs tend to diminish the magnitude of any change in actions. This effect vanishes over time as the agent completes all of the desired adjustments, so it tends to make actions less responsive to weather variation than to changes in climate.[31] The biases introduced by Ω and ω therefore conflict.

Now consider resource scarcity stories, which have $\bar{\pi}_{12} < 0$. The second term in parentheses now reflects how the agent conserves resources for tomorrow by shifting today's actions away from the level that will be chosen tomorrow ($\Omega < 0$). This effect again vanishes as time passes and actions become more similar between today and tomorrow, so it tends to make actions less responsive to weather variation than to changes in climate. The $\omega > 1$ captures how resource scarcity tend to allow for more extreme actions when the actions will be maintained for only a short period of time. As a result, actions are more responsive to weather variation than to changes in climate. Once again, the two biases conflict.

Figure 2 illustrates the intuition. Begin with the left panel, which is a case of adjustment costs. For exposition, ignore the possibility of ex-ante adaptation or delayed effects. As in the left panel of Figure 1, the solid curves depict time t payoffs conditional on time t actions,

[31]This effect has the flavor of Le Châtelier's principle.

with point a indicating an action chosen in a typical period and point b indicating an action chosen in a hotter period. The difference between points a and b is controlled by $\bar{\pi}_{13}$ and reflects the effects of current weather in equation (7). Also as in the left panel of Figure 1, the dashed curve reflects payoffs conditional on both the current period being hot and the past period having been hot. When the previous period was hot, the agent chose higher actions than otherwise. Those past choices of high actions reduce the cost of choosing high actions in the current period. The current period's choice of action therefore increases to point c. The difference between points b and c is controlled by $\bar{\pi}_{12}$ and reflects the effects of past weather in equation (7). Finally, point d reflects the implications of expecting the subsequent period to again be hot and thus of expecting to choose a high action in the subsequent period.[32] The marginal benefit of choosing a high action in the current period increases because high current actions reduce future adjustment costs. The agent's optimal action therefore increases to point d, where the marginal effect on current-year payoffs is negative. The difference between points c and d is controlled by $\beta\bar{\pi}_{13}\bar{\pi}_{12}$ and reflects the effects of future weather in equation (7).

The action that will be desired in a subsequent period is more different from the current period's action when the high forecast reflects a transient shock. In that case, the adjustment will be larger and the current period's action increases to an even larger point, represented by point e. The change to point e reflects $\Omega > 0$ and illustrates how agents may respond more strongly to transient weather shocks. In addition, past actions are lower following a transient shock to past weather than they would be if a longer history of weather had changed as a result of a shift in climate. The resulting adjustment costs bring payoffs closer to the rightmost solid curve than to the dashed curve. Those adjustment costs reduce actions to a point such as f. The change to point f reflects $\omega \in (0, 1)$ and illustrates how agents may respond less strongly to transient weather shocks. It is difficult in general to determine whether point f is to the left or to the right of point d because point f results from the combination of a rightward shift from point d to point e and a leftward shift from point e to point f.

The right panel of Figure 2 depicts a resource scarcity story. The solid curves and points a and b are as before, and the dashed curve again depicts a case in which the previous period was also hot and thus saw the agent choose high actions. However, that dashed curve has now shifted down because the previous period's high actions increase the cost of the current period's actions by having used scarce resources such as groundwater or time. Point c is therefore now to the left of point b. Point d again reflects the expectation of the subsequent period being hot, but it is now shifted to the left of point c because the expectation of high actions in the subsequent period increases the benefit of freeing up resources by choosing

[32] For exposition, I have plotted points a, b, and c as occurring at the myopically optimal actions, where $\pi_1 = 0$. However, as described around equation (1), these points could in general have $\pi_1 \neq 0$. In either case, point d would be to the right of point c under the given story.

low actions in the current period.[33] If the forecasted event is transient, then the subsequent period's action will be especially different from the current period's action and the benefit of freeing up resources that much larger. Point e is therefore now to the left of point d, reflecting $\Omega < 0$ and illustrating how agents may respond less to transient weather shocks. However, past actions are also lower following transient weather shocks than following a shift in climate, so resources are not as scarce in the current period if climate change has not yet occurred. Point f therefore is now to the right of point e, reflecting $\omega > 1$ and illustrating how agents may respond more to transient weather shocks. It is difficult in general to determine whether point f is to the left or to the right of point d because point f results from the combination of a leftward shift from point d to point e and a rightward shift from point e to point f.

The following proposition establishes special cases in which the biases vanish and in which their net effect can be clearly signed.

Proposition 2. *Let either Assumption 1 or 2 hold, and let $(A_{j(t-1)} - \bar{A})^2$ be small for all observations.*

1. *Testing Intertemporal Substitutes/Complements: $\bar{\pi}_{12} < 0$ if and only if $\hat{\Gamma}_4 < 0$.*
2. *Independence From Past Actions: If $\bar{\pi}_{12} = 0$, then (i) $\hat{\Gamma}_1 + \hat{\Gamma}_2 + \hat{\Gamma}_3 = \mathrm{d}\bar{A}/\,\mathrm{d}C$ and (ii) $\hat{\Gamma}_4 = 0$.*
3. *No Ex-Ante Adaptation or Delayed Effects: If $\bar{\pi}_{23} = 0$, then $\bar{\pi}_{12} < 0$ if and only if $\hat{\Gamma}_3 < 0$. If, in addition, $\bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\bar{\pi}_{13} > 0$, then $\hat{\Gamma}_1 > \mathrm{d}\bar{A}/\,\mathrm{d}C$ if and only if $\bar{\pi}_{12} < 0$.*
4. *No Ex-Post Adaptation: If $\bar{\pi}_{13}, \bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\beta\bar{\pi}_{23} > 0$, then (i) $\hat{\Gamma}_3 > \mathrm{d}\bar{A}/\,\mathrm{d}C$ if and only if $\bar{\pi}_{12} < 0$ and (ii) $\hat{\Gamma}_1 = \hat{\Gamma}_2 = 0$.*
5. *Myopic Agents: If $\beta = 0$, then $\hat{\Gamma}_3 = 0$. If, in addition, either $\bar{\pi}_{13} > 0$ or $\bar{\pi}_{14} > 0$, then $\hat{\Gamma}_1 + \hat{\Gamma}_2 > \mathrm{d}\bar{A}/\,\mathrm{d}C$ if and only if $\bar{\pi}_{12} < 0$.*

Proof. See appendix. □

The first result says that we can learn whether actions are intertemporal substitutes or complements by considering the coefficient on previous actions. The second result confirms that we can exactly recover the effect of climate from transient weather shocks when that coefficient is zero. The intuition should be clear from the foregoing discussion: both sources of bias vanish when $\bar{\pi}_{12} = 0$, with $\omega = 1$ and $\Omega = 0$.

[33] In general, point d could be to the right or the left of point c because the effects of $\beta\bar{\pi}_{23}$ conflict with the channel discussed here, but I am temporarily ignoring the possibility of ex-ante adaptation.

The remaining results describe cases in which we cannot exactly recover the response to a long-run change in climate but can sign the bias in our estimate. The third result describes a case without either ex-ante adaptation or delayed effects, so that only current weather matters and people respond to that weather only as it happens. This case may adequately describe household decisions about how to set a thermostat. If there is no ex-ante adaptation, then forecasts matter only because they shape expectations of future actions, not because they allow actions that can directly interact with future weather. And if there are no delayed effects, then we do not need to concern ourselves with the direct consequences of past weather. $\hat{\Gamma}_1$ then captures all the channels of interest and is distorted only via the ω in equation (10).

In contrast, the fourth result considers a case without ex-post adaptation. Here adaptation actions require a lead time, as with the decision to buy an air conditioning unit or the decision about which crop to plant. In this case, current weather should have no effect on observed actions and the effects of climate change arise only through altered expectations, not through effects on either current or past weather. $\hat{\Gamma}_3$ then captures all the channels of interest and is distorted only via the ω in equation (10).

Some previous literature has highlighted expectations as being the sole difference between weather and climate (e.g., Hsiang, 2016; Deryugina and Hsiang, 2017), but the final result shows that differences remain even for myopic agents, for whom expectations are irrelevant. Forecasts do not matter to myopic agents ($\hat{\Gamma}_3 = 0$), but their responses to weather still fail to recover the effect of climate change. The reason is that their current actions do depend on past actions, even though they fail to anticipate this dependence. This dependence constrains short-run responses when actions are intertemporal complements and constrains long-run responses when actions are intertemporal substitutes. Formally, setting $\beta = 0$ in equation (10) eliminates the bias introduced by Ω but does not eliminate the bias introduced by ω.

We have developed intuition and results for a regression like (9), but the empirical literature has, almost without exception, not estimated that type of specification. Instead, researchers have often estimated regressions of the form

$$A_{jt} = \alpha_j + \psi_t + \gamma_1 w_{jt} + \gamma_2 w_{j(t-1)} + \delta_{jt}. \tag{11}$$

Note that the error term δ_{jt} is now correlated with the covariates because it includes f_{jt} and $A_{j(t-1)}$.[34] We then have:

[34] One could in principle include $A_{j(t-1)}$ as a control on the right-hand side of'(11), but standard advice (e.g., Angrist and Pischke, 2009, Chapter 5) recommends against controlling for both fixed effects and lagged dependent variables because of the likelihood of introducing Nickell (1981) omitted variables bias. Standard practice in the empirical climate-economy literature has emphasized fixed effects instead of lagged dependent variables, as seen in the appendix to Dell et al. (2014). Controlling for $A_{j(t-1)}$ would not change the primary results in Proposition 3.

Proposition 3. *Let either Assumption 1 or 2 hold, and let $(A_{j(t-1)} - \bar{A})^2$ be small for all observations.*

1. *Fix $\bar{\pi}_{12} = 0$ and consider $\hat{\gamma}_1$:*
   (a) $\hat{\gamma}_1 = \mathrm{d}\bar{A}/\,\mathrm{d}C$ *if* $\bar{\pi}_{14} = 0$ *and* $\beta\bar{\pi}_{23} = 0$.
   (b) $\hat{\gamma}_1 \in (\hat{\Gamma}_1, \mathrm{d}\bar{A}/\,\mathrm{d}C)$ *if* $\rho\beta\bar{\pi}_{23} > 0$.
   (c) $\hat{\gamma}_1 = \hat{\Gamma}_1$ *if* $\rho\beta\bar{\pi}_{23} = 0$.
2. *Fix $\bar{\pi}_{12} = 0$ and consider $\hat{\gamma}_1 + \hat{\gamma}_2$:*
   (a) $\hat{\gamma}_1 + \hat{\gamma}_2 = \mathrm{d}\bar{A}/\,\mathrm{d}C$ *if* $\beta\bar{\pi}_{23} = 0$.
   (b) $\hat{\gamma}_1 + \hat{\gamma}_2 \in (\hat{\Gamma}_1 + \hat{\Gamma}_2, \mathrm{d}\bar{A}/\,\mathrm{d}C)$ *if* $\rho\beta\bar{\pi}_{23} > 0$.
   (c) $\hat{\gamma}_1 + \hat{\gamma}_2 = \hat{\Gamma}_1 + \hat{\Gamma}_2$ *if* $\rho\beta\bar{\pi}_{23} = 0$.
3. *If $\beta = 0$, then $\hat{\gamma}_1 < \mathrm{d}\bar{A}/\,\mathrm{d}C$ if $\bar{\pi}_{12} > 0$. If, in addition, $\bar{\pi}_{14} = 0$, then $\hat{\gamma}_1 > \mathrm{d}\bar{A}/\,\mathrm{d}C$ if $\bar{\pi}_{12} < 0$.*

Proof. See appendix. □

Regression (11) does not control for forecasts or for past actions, so these affect the estimators $\hat{\gamma}_1$ and $\hat{\gamma}_2$ as omitted variables. The first set of results establishes what we can learn from $\hat{\gamma}_1$, which is the coefficient of interest in much previous empirical literature. Assume that the marginal benefit of current actions is independent of past actions ($\bar{\pi}_{12} = 0$). $\hat{\gamma}_1$ captures part of the effect of time t forecasts through their covariance ρ with ϵ_t; however, the proof shows that $\hat{\gamma}_1$ can never capture the total effect of forecasts. Omitted variables bias helps, but it cannot replace explicitly controlling for forecasts. Further, $\hat{\gamma}_1$ also misses the interaction between time t actions and past weather. Putting these pieces together, $\hat{\gamma}_1$ can fully recover climate impacts only if there is no ex-ante adaptation that would use forecasts ($\beta\bar{\pi}_{23} = 0$) and past weather shocks do not affect actions directly ($\bar{\pi}_{14} = 0$). In other cases, $\hat{\gamma}_1$ underestimates the effect of climate change by only partially capturing these channels. The performance of $\hat{\gamma}_1$ improves as weather becomes more serially correlated (i.e., as ρ increases) because omitted variables bias becomes stronger, but one could do better by estimating equation (9) and combining $\hat{\Gamma}_1$ with $\hat{\Gamma}_2$ and $\hat{\Gamma}_3$.

The second set of results shows that combining $\hat{\gamma}_1$ and $\hat{\gamma}_2$ captures the interaction between time t actions and past weather but still fails to capture the total effect of forecasts. Omitted variables bias can make $\hat{\gamma}_1 + \hat{\gamma}_2$ perform better than $\hat{\Gamma}_1 + \hat{\Gamma}_2$, but if weather is serially uncorrelated ($\rho = 0$), then omitted variables bias from forecasts vanishes and these two estimators are equivalent. And researchers could always do better by estimating equation (9) and combining $\hat{\Gamma}_1 + \hat{\Gamma}_2$ with $\hat{\Gamma}_3$.

The final result again shows that expectations are not the only factor driving a wedge between weather and climate. When actors are myopic ($\beta = 0$) and current actions are independent of previous weather ($\bar{\pi}_{14} = 0$), climate directly matters for decision-making only by affecting present weather. However, the effect of climate on past weather matters indirectly even in this case by shaping past actions. When actions are intertemporal substitutes ($\bar{\pi}_{12} < 0$), these past actions constrain the long-run response to climate more than the response to transient weather events, but when actions are intertemporal complements ($\bar{\pi}_{12} > 0$), these past actions constrain the response to transient weather events more than the long-run response to climate.

5 Estimating the Effect of Climate on Payoffs

Now return to regression (6). We previously analyzed this regression using Assumption 3, but we are now interested in whether it can recover the effects of climate when Assumption 3 does not hold.

Proposition 4. *Let Assumption 1 hold, or let Assumption 2 hold with the ϵ and ν normally distributed. Also let $(A_{j(t-1)} - \bar{A})^2$ and $(A_{jt} - \bar{A})^2$ be small for all observations and let each agent's average actions be $\bar{A}$. Then, for s large and $K > 2$,*

1. *The Estimators: $\hat{\theta}_{w_t} = \bar{\pi}_3 + \bar{\pi}_1 \hat{\Gamma}_1$ and $\hat{\theta}_{f_t} = \bar{\pi}_1 \hat{\Gamma}_3$. If $\bar{\pi}_{12} = 0$, then $\hat{\theta}_{w_{t-1}} = \bar{\pi}_4 + \bar{\pi}_2 \hat{\Gamma}_1 + \bar{\pi}_1 \hat{\Gamma}_2$, $\hat{\theta}_{f_{t-1}} = \bar{\pi}_2 \hat{\Gamma}_3$, and $\hat{\theta}_{w_{t-2}} = \bar{\pi}_2 \hat{\Gamma}_2$.*

2. *Independence From Past Actions: If $\bar{\pi}_{12} = 0$, then $\mathrm{d}E_0[\pi_s]/\,\mathrm{d}C = \hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}}$.*

3. *No Ex-Ante Adaptation or Delayed Effects: Let $\bar{\pi}_{23}, \bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\bar{\pi}_{13} > 0$. Then $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} < \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ if and only if $\bar{\pi}_2 \bar{\pi}_{12} > 0$.*

4. *No Ex-Post Adaptation: Let $\bar{\pi}_{13}, \bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\beta \bar{\pi}_{23} > 0$. Then $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}} < \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ if and only if $\bar{\pi}_2 \bar{\pi}_{12} > 0$.*

5. *Myopic Agents: If $\beta = 0$, then $\hat{\theta}_{f_t} = \hat{\theta}_{f_{t-1}} = 0$. If, in addition, either $\bar{\pi}_{13} > 0$ or $\bar{\pi}_{14} > 0$, then $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} < \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ if and only if $\bar{\pi}_2 \bar{\pi}_{12} > 0$.*

Proof. See appendix. □

From equation (4), estimating the effects of climate on payoffs requires estimating four terms: $\bar{\pi}_3$, $\bar{\pi}_4$, $\bar{\pi}_1[\,\mathrm{d}\bar{A}/\,\mathrm{d}C]$, and $\bar{\pi}_2[\,\mathrm{d}\bar{A}/\,\mathrm{d}C]$. The first result of the proposition shows that the coefficients from (6) are closely related to these terms. The second result shows that we can recover the effect of climate on payoffs when $\bar{\pi}_{12} = 0$. From Proposition 2, we know that $\hat{\Gamma}_1 + \hat{\Gamma}_2 + \hat{\Gamma}_3$ recovers the effect of climate on actions when $\bar{\pi}_{12} = 0$. The first part of Proposition 4 showed that $\hat{\theta}_{w_t}$ and $\hat{\theta}_{w_{t-1}}$ recover $\hat{\Gamma}_1$ (the consequences of altering current weather), $\hat{\theta}_{w_{t-1}}$ and $\hat{\theta}_{w_{t-2}}$ recover $\hat{\Gamma}_2$ (the consequences of altering past weather), and $\hat{\theta}_{f_t}$ and $\hat{\theta}_{f_{t-1}}$ recover $\hat{\Gamma}_3$ (the consequences of altering expectations of future weather). The coefficient in each pair with the larger time index captures the effect on payoffs via current actions and the other coefficient captures the effect on payoffs via past actions. Therefore, summing $\hat{\theta}_{w_t}$, $\hat{\theta}_{w_{t-1}}$, $\hat{\theta}_{w_{t-2}}$, $\hat{\theta}_{f_t}$, and $\hat{\theta}_{f_{t-1}}$ recovers $(\bar{\pi}_1 + \bar{\pi}_2)[\,\mathrm{d}\bar{A}/\,\mathrm{d}C]$ when $\bar{\pi}_{12} = 0$. When neither adjustment cost nor resource scarcity stories apply, we can recover the effect of climate on payoffs from a regression with sufficiently long lags of weather and forecasts.[35]

The third part of the proposition signs the bias in our estimate of the effect of climate on payoffs when $\bar{\pi}_{12} \neq 0$ but there is no ex-ante adaptation or delayed effects. In this case, only current weather matters for actions. This channel is captured by $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}}$. From Proposition 2, we know that adjustment costs and resource scarcity each lead us to misestimate effects on actions: $\hat{\Gamma}_1 > \,\mathrm{d}\bar{A}/\,\mathrm{d}C$ if and only if $\bar{\pi}_{12} < 0$. Equation (5) showed that undertaking more actions because of climate change provides benefits if and only if $\bar{\pi}_2 > 0$. Therefore, $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}}$ is an overly pessimistic estimate of the effects of climate on payoffs in two cases: when actions are beneficial and variation in weather underestimates how actions respond to climate change ($\bar{\pi}_2, \bar{\pi}_{12} > 0$), and when actions impose costs and actions respond less to climate change than to short-lived changes in weather ($\bar{\pi}_2, \bar{\pi}_{12} < 0$). Intuitively, a case with $\bar{\pi}_2, \bar{\pi}_{12} > 0$ is one in which actions are costly to adjust and are undertaken for future benefits, as with diverting crops to storage, and a case with $\bar{\pi}_2, \bar{\pi}_{12} < 0$ is one in which actions impose long-run costs but will not be maintained for long, as may be true of groundwater withdrawals. In either case, extrapolating from responses to weather overstates the cost of climate change; in other cases, the estimator $\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}}$ is an overly optimistic estimate of the effect of climate change.

The fourth part of the proposition signs the bias in our estimate of the effect of climate on payoffs when $\bar{\pi}_{12} \neq 0$ but there is no ex-post adaptation. In this case, only expectations of future weather matter for actions. This channel is is captured by $\hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}}$. From Proposition 2, we know that adjustment costs and resource scarcity lead us to misestimate effects on actions: $\hat{\Gamma}_3 > \,\mathrm{d}\bar{A}/\,\mathrm{d}C$ if and only if $\bar{\pi}_{12} < 0$. From here, the intuition for the result follows the case without ex-ante adaptation or delayed effects.

The final result considers a case with myopic agents. When agents are myopic, the Euler equation (1) implies that $\bar{\pi}_1 = 0$. The effects of climate on current actions no longer matter for payoffs, but the effects of climate on past actions can still matter for payoffs.[36]

[35] The requirement that $K > 2$ ensures that omitted variables bias does not affect the needed coefficients. If we allowed for longer lags in the consequences of weather and/or in the consequences of forward-looking investments, then the estimator of climate consequences would include longer lags of forecasts and weather than the estimator described in part 2 of Proposition 4. We would then require that K be strictly larger than the longest lag.

[36] Because myopic agents solve a static decision problem, the envelope theorem now makes current actions independent of payoffs. But a static decision problem is not equivalent to a static decision-making environment: because past actions are predetermined variables, the envelope theorem has no bearing on how the decision problems are linked through the history of weather. In Figures 1 and 2, even myopic agents find

Proposition 2 already established that we tend to underestimate changes in actions when $\bar{\pi}_{12} > 0$. In this case, we estimate an overly pessimistic effect of climate on payoffs if and only if past actions provide current benefits ($\bar{\pi}_2 > 0$). On the other hand, we tend to overestimate the changes in actions when $\bar{\pi}_{12} < 0$. In this case, we estimate an overly optimistic effect of climate on payoffs if and only if past actions impose current costs ($\bar{\pi}_2 < 0$). Stripping away expectations does not eliminate all of the dynamic linkages that differentiate climate from weather.

Propositions 1 and 4 assumed that each agent's average actions are $\bar{A}$. The following corollary establishes how relaxing this assumption changes the results.

Corollary 5. *Let the conditions given in Proposition 4 hold, except let each agent's average actions be strictly greater than $\bar{A}$. Then, for s large and $K > 2$,*

1. $\hat{\theta}_{f_t} = \bar{\pi}_1 \hat{\Gamma}_3$ *and, if either* $\bar{\pi}_{13} > 0$ *or* $\bar{\pi}_{23} > 0$, $\hat{\theta}_{w_t} > \bar{\pi}_3 + \bar{\pi}_1 \hat{\Gamma}_1$. *If* $\bar{\pi}_{12} = 0$, *then* $\hat{\theta}_{f_{t-1}} = \bar{\pi}_2 \hat{\Gamma}_3$, $\hat{\theta}_{w_{t-2}} = \bar{\pi}_2 \hat{\Gamma}_2$, *and, if either* $\bar{\pi}_{14} > 0$ *or* $\bar{\pi}_{24} > 0$, $\hat{\theta}_{w_{t-1}} > \bar{\pi}_4 + \bar{\pi}_2 \hat{\Gamma}_1 + \bar{\pi}_1 \hat{\Gamma}_2$.

2. *If Assumption 3 holds and at least one of* $\bar{\pi}_{13}$, $\bar{\pi}_{23}$, $\bar{\pi}_{14}$, *or* $\bar{\pi}_{24}$ *is strictly positive, then* $\mathrm{d}E_0[\pi_s]/\,\mathrm{d}C = \bar{\pi}_3 + \bar{\pi}_4 < \hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}}$.

3. *If* $\bar{\pi}_{12} = 0$ *and at least one of* $\bar{\pi}_{13}$, $\bar{\pi}_{23}$, $\bar{\pi}_{14}$, *or* $\bar{\pi}_{24}$ *is strictly positive, then* $\mathrm{d}E_0[\pi_s]/\,\mathrm{d}C < \hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}}$.

The inequalities reverse if, instead, each agent's average actions are strictly less than $\bar{A}$.

Proof. See appendix. □

The first part of the corollary establishes that the bias from average actions not yet having reached the steady state enters through $\hat{\theta}_{w_t}$ and $\hat{\theta}_{w_{t-1}}$. The remaining parts of the corollary establish that the special cases that formerly sufficed to identify climate impacts from weather impacts now merely bound the effect of climate on payoffs. In particular, we obtain an upper bound if agents are approaching their steady-state actions from above and a lower bound otherwise. Intuitively, if climate shifts the steady-state action farther from the agent's current action, then any weather shocks incorporate transition costs that vanish from the effect of climate on long-run payoffs.

In practice, empirical researchers have not estimated equations like (6). Instead, empirical researchers have rarely controlled for forecasts and often have not controlled for lagged weather outcomes.[37] Conventional regressions are closer to

$$\pi_{jt} = \alpha_j + \psi_t + X_{jt}\Phi + \delta_{jt}, \tag{12}$$

point c instead of remaining at point b following a weather shock.

[37] Distributed lag models are more commonly estimated when mortality is the dependent variable (e.g., Deschênes and Moretti, 2009) than when payoffs are the dependent variable.

where the vector of covariates X_{jt} is now

$$\begin{bmatrix} w_{jt} & w_{j(t-1)} & \dots & w_{j(t-K)} \end{bmatrix}.$$

The error term δ_{jt} now includes not only actions but also current and past forecasts. We are interested in the vector of coefficients Φ. I denote each element with a subscript corresponding to the covariate it multiplies, and I again use a hat to denote the probability limit of each element. A superscript on each $\hat{\Phi}$ now denotes the value of K.

The following proposition relates these estimates to the desired effect of climate on payoffs.

Proposition 6. *Let Assumption 1 hold, or let Assumption 2 hold with the ϵ and ν normally distributed. Also let $(A_{j(t-1)} - \bar{A})^2$ and $(A_{jt} - \bar{A})^2$ be small for all observations, let each agent's average actions be $\bar{A}$, and let s be large.*

1. *Let Assumption 3 hold. Then:*
   (a) $\hat{\Phi}^K_{w_t} + \hat{\Phi}^K_{w_{t-1}} = \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ *if* $K = 1$ *or* $K = 2$.
   (b) $\hat{\Phi}^0_{w_t} < \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ *if and only if* $\bar{\pi}_4 > 0$.
2. *Let $\bar{\pi}_{12} = 0$ and $\beta\bar{\pi}_{23} = 0$. Then:*
   (a) $\hat{\Phi}^2_{w_t} + \hat{\Phi}^2_{w_{t-1}} + \hat{\Phi}^2_{w_{t-2}} = \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$.
   (b) *If* $\bar{\pi}_{14} = 0$, *then* $\hat{\Phi}^1_{w_t} + \hat{\Phi}^1_{w_{t-1}} = \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$.
   (c) *If* $\bar{\pi}_4 = 0$ *and* $\bar{\pi}_{13} > 0$, *then* $\hat{\Phi}^0_{w_t} < \mathrm{d}E_0[\pi_s]/\,\mathrm{d}C$ *if and only if* $\bar{\pi}_2 > 0$.

Proof. See appendix. □

The first result establishes that we can still recover the full effect of climate on payoffs if Assumption 3 holds and $K > 0$. Responses to forecasts generally identify ex-ante adaptation, but small changes in adaptation are irrelevant for payoffs when Assumption 3 holds. And Assumption 3 further implies that responses to forecasts do not induce omitted variables bias in (12). We can therefore recover the effect of climate from $\hat{\Phi}^K_{w_t} + \hat{\Phi}^K_{w_{t-1}}$.

However, the second result establishes that $\bar{\pi}_{12} = 0$ no longer suffices to recover the effects of climate on payoffs. Because regression (12) fails to control for forecasts, the estimated coefficients no longer fully capture the possibility of ex-ante adaptation based on expectations of future weather.[38] We must therefore introduce a further assumption—that there is no ex-ante adaptation ($\beta\bar{\pi}_{23} = 0$)—in order to recover the effect of climate. If we estimate a model

[38]Some of this ex-ante adaptation is captured through omitted variables bias in the plausible case where weather and forecasts are positively correlated (i.e., where $\rho > 0$), but the proof shows that it can never be captured completely.

with only a single lag of weather, then we must also limit the interaction between actions and past weather by assuming that $\bar{\pi}_{14} = 0$. And if we follow much of the empirical literature in estimating a model without any lags of weather, then recovering the effects of climate change further requires the absence of adaptation to current weather ($\bar{\pi}_{13} = 0$) and the absence of any lagged effects of weather ($\bar{\pi}_4 = 0$). These strong assumptions are unlikely to hold in many applications of interest.[39]

6 Aggregating over Longer Timesteps

In actual empirical work, the proper timestep of analysis may be unclear, computational requirements may require using coarser timesteps, or data may be available only over coarser timesteps. We therefore now consider the implications of aggregating weather and payoffs over longer timesteps.

Assume, as before, that all agents are in the same climate and that this climate is stationary. The empirical researcher averages outcomes over Δ periods.[40] Denote the averages with a tilde and use the time subscript to indicate the beginning of the averaging interval, so that, for instance, $\tilde{\pi}_{jt} \triangleq \sum_{T=t}^{t+\Delta-1} \pi_{jT}/\Delta$. Consider estimating the regression

$$\tilde{\pi}_{jt} = \Lambda \tilde{w}_{jt} + \tilde{u}_{jt},$$

where I assume that the averaging intervals do not overlap and that $\tilde{u}_{jt}$ is uncorrelated with $\tilde{w}_{jt}$.

The following proposition establishes properties of the estimator $\hat{\Lambda}$.

Proposition 7. *Let Assumption 1 hold, or let Assumption 2 hold with the ϵ and ν normally distributed. Also let $(A_{j(t-1)} - \bar{A})^2$ and $(A_{jt} - \bar{A})^2$ be small for all observations and let each agent's average actions be $\bar{A}$. Then the following conditions are individually sufficient for $\mathrm{d}E_0[\pi_s]/\mathrm{d}C \to \hat{\Lambda}$ as $s, \Delta \to \infty$:*

1. *Assumption 3 holds.*

[39]Rather than focusing on the $\hat{\Phi}$, Deryugina and Hsiang (2017) undertake a different calculation. Let $p(w_t; C)$ represent the probability density function for weather in climate C. They estimate $\pi(A_t(w_t), A_{t-1}(w_t), w_t, w_{t-1}(w_t)) - \pi(A_t(w^0), A_{t-1}(w^0), w^0, w_{t-1}(w^0))$ for each w_t, where w^0 indicates the omitted category and where we write $A_{t-1}(w_t)$ and $w_{t-1}(w_t)$ in order to focus on questions besides the evaluation point. They calculate the marginal effect of climate from the following expression: $\int_{-\infty}^{\infty} \left[\pi(A_t, A_{t-1}, w_t, w_{t-1}) - \pi(A_t, A_{t-1}, w^0, w_{t-1})\right] \frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C} \mathrm{d}w_t \triangleq \Psi$. Analyzing, we find $\Psi = Cov\left[\pi(A_t, A_{t-1}, w_t, w_{t-1}), \frac{\frac{\mathrm{d}p(w_t;C)}{\mathrm{d}C}}{p(w_t;C)}\right]$. If w_t is normally distributed, then $\Psi = Cov\left[\pi(A_t, A_{t-1}, w_t, w_{t-1}),\ w_t\right]/Var[w_t]$, which, following the proof of Proposition 6, is equal to $\hat{\Phi}^0_{w_t}$. Proposition 6 shows that this estimator recovers the effects of climate change in only the most special of cases.

[40]The results do not depend on whether the operation is averaging or summing.

2. $\rho, \bar{\pi}_{12}, \beta\bar{\pi}_{23} = 0$.

3. $\rho, \bar{\pi}_{12} = 0$ *and* $\sigma^2/\tau^2 = 0$.

Proof. See appendix. □

The proof shows that, for $\Delta > 2$, the estimator is

$$\hat{\Lambda} = \hat{\Phi}^0_{w_t} + \frac{\Delta - 1}{\Delta}\Upsilon_1 + \frac{\Delta - 2}{\Delta}\Upsilon_2,$$

with $\Upsilon_1, \Upsilon_2 \geq 0$. As Δ becomes small, the long-timestep estimator $\hat{\Lambda}$ converges towards $\hat{\Phi}^0_{w_t}$, which Proposition 6 showed can approximate the effect of climate change in only the most special of cases. As Δ becomes large, the coefficients on Υ_1 and Υ_2 go to 1. This is the case considered by Proposition 7. As Δ becomes large, $\hat{\Lambda}$ recovers the effect of climate change in a broader set of cases than does $\hat{\Phi}^0_{w_t}$: the process of aggregating Δ time periods into one picks up correlations between current payoffs and lags and leads of weather within these Δ periods. However, $\hat{\Lambda}$ underperforms estimators analyzed in Section 3. In particular, for $\rho, \bar{\pi}_{12} = 0$ and Δ large, the estimator $\hat{\Lambda}$ recovers the effect of climate only in the absence of ex-ante adaptation, which was the same restriction required by the estimator $\hat{\Phi}^2_{w_t} + \hat{\Phi}^2_{w_{t-1}} + \hat{\Phi}^2_{w_{t-2}}$ studied in Proposition 6.[41]

The estimator $\hat{\Lambda}$ is therefore closely to the panel regression (12). This insight has implications for a recent literature developing "long difference" estimates of climate impacts. Rather than estimating either a cross-sectional or a panel model, this method instead averages weather and outcomes over two non-overlapping periods, differences the averages, and estimates how the differenced dependent variable changes with differenced average weather (e.g., Dell et al., 2012; Burke and Emerick, 2016). To many, this approach's appeal rests in providing "plausibly credible causal estimates of climate impacts that account for adaptation" (Auffhammer, 2018b, 45): differencing removes the unobserved fixed factors that may covary with climate in a cross-sectional regression, and the variation induced by spatially heterogeneous rates of climate change may identify the long-run adaptations missing from standard panel regressions. On this reasoning, comparing long difference estimates to standard panel estimates indicates whether short-run adaptation differs from long-run adaptation.

In the present setting, there is no climate change (C is constant over time), yet it is easy to show that the estimator $\hat{\Lambda}$ is equivalent to a long difference estimator. Proposition 7

[41] When there is ex-ante adaptation, the third case in Proposition 7 shows that $\hat{\Lambda}$ improves on $\hat{\Phi}^2_{w_t} + \hat{\Phi}^2_{w_{t-1}} + \hat{\Phi}^2_{w_{t-2}}$ as Δ becomes large. This improvement arises because $\hat{\Lambda}$ picks up correlation between payoffs and leads of weather, which accounts for more ex-ante adaptation than does $\hat{\Phi}^2_{w_t} + \hat{\Phi}^2_{w_{t-1}} + \hat{\Phi}^2_{w_{t-2}}$ as Δ becomes large.

therefore implies that long difference estimators are in fact identified by random differences in sequences of transient weather shocks over the aggregation intervals. At best, the long difference estimator conflates this variation with differential rates of climate change, but at worst, it captures nothing but the same transient weather shocks as do the estimators in (12).[42] Intuitively, averaging over several periods does not eliminate the old sources of variation and need not introduce new sources of variation. In fact, previous work has found that long difference and panel estimators produce similar results (Burke and Emerick, 2016). We now see that this result should be unsurprising: rather than indicating the absence of long-run adaptation, the similarity may in fact be mechanical.

7 Caveats and Potential Extensions

I have demonstrated how to estimate the effects of climate change from time series variation in weather. The setting is fairly general. Nonetheless, the results are subject to several caveats.

First, the present setting successfully captures the distinction between transient and permanent changes in weather, but global climate change also differs from most weather shocks in its spatial structure. A change in global climate affects weather in every location and thus will have general equilibrium consequences. The present setting has followed most empirical work in abstracting from such effects, but some recent empirical work has begun exploring the implications of changing the weather in many locations simultaneously (e.g., Costinot et al., 2016; Gouel and Laborde, 2018; Dingel et al., 2019). Future work should extend the present setting to account for general equilibrium effects.

Second, the present analysis has held the payoff function fixed over time. However, climate change should induce innovations that alter how weather affects payoffs, and many such innovations will arise even in the absence of climate change. Some types of innovation can be interpreted as actions within the present framework, but estimating future innovation poses distinct challenges. Historical studies have begun exploring the interaction between climate and agricultural innovation (e.g., Olmstead and Rhode, 2008, 2011; Roberts and Schlenker, 2011; Bleakley and Hong, 2017), but the potential for future innovation may be inherently unobservable. Future work should consider approaches to bounding the scope for innovation.

Third, the present analysis has considered only marginal changes in climate, but climate change over the next century is likely to be nonmarginal. One could approximate the consequences of nonmarginal changes in climate by summing the estimates from fixed effects regressions undertaken in different climate zones. Time series variation then identifies

[42] That worst case arises when the climate actually did not change differentially (as modeled here) or when agents were not aware of ongoing changes in climate. See Dell et al. (2014) and Burke and Emerick (2016) for discussion of awareness of climate change.

the consequences of marginal changes in climate and cross-sectional variation identifies the consequences of nonmarginal changes in climate. Similar approaches to combining panel and cross-sectional variation have recently been summarized by Auffhammer (2018b). However, two considerations call for caution when extrapolating reduced-form estimates to large changes in climate: the use of cross-sectional variation raises the usual concerns about identification, which becomes more severe as that cross-sectional variation is asked to do more work, and higher-order effects are likely to become relevant to nonmarginal climate change, even though absent from a summation of estimated marginal effects. Future work should explore whether nonlinear responses to weather shocks can inform nonmarginal consequences of climate change.[43]

Fourth, the present analysis has ignored the possibility of fixed costs to changing actions. In the presence of fixed costs, an agent may choose to change an action only when the agent expects a change in weather to endure.[44] Future work should explore the conditions under which aggregating over many agents' fixed-cost decisions makes actions appear continuous. Future work should also explore whether the duration of forecasts or realized weather events can identify how fixed cost actions respond to a change in climate.

Finally, the present analysis has focused on identifying the long-run consequences of climate change, abstracting from the transition costs induced by climate change. In this regard, the present analysis matches the calculations undertaken by nearly all empirical work but omits a potentially critical aspect of climate change (see Quiggin and Horowitz, 1999, 2003; Kelly et al., 2005). Future work should consider whether imposing stronger assumptions on the decision-making environment can identify the costs of full-information transitions. Further, the project of the paper has been to study the potential for reduced-form empirical work to recover the effects of climate, but future work should consider the potential to estimate structural models that could credibly simulate outcomes along counterfactual climate trajectories. The present analysis suggests that such structural models could be calibrated to replicate reduced-form estimates of the effects of marginal climate change.

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[43]Estimating the consequences of nonmarginal climate change is critical to the damage functions required by integrated assessment models. However, there is an argument that the consequences of marginal climate change might be more policy relevant: if we accept climate scientists' views that the risks imposed by nonmarginal climate change are likely to exceed the cost of avoiding them, then the costs (or benefits) of marginal climate change becomes critical to policy choices.

[44]In a related analysis, Guo and Costello (2013) explore the implications of restricting actions to a discrete choice set.

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A Proof of Lemma 1

The case with $\pi_{12} = 0$ is trivial. I therefore analyze cases with $\pi_{12} \neq 0$.

Begin by considering the uniqueness of the steady state. The right-hand side of equation (2) monotonically decreases in $\bar{A}$ if and only if $(1+\beta)\pi_{12} < -\pi_{11} - \beta\pi_{22}$. Thus, the steady state is unique if $(1+\beta)\pi_{12} < -\pi_{11} - \beta\pi_{22}$, which is satisfied for all $\pi_{12} < 0$.

Now consider the stability of the steady state. Define $A^*_{t+1}(A_t, A_{t-1})$ from the Euler equation. Linearizing around $\bar{A}$ gives a first-order difference equation:

$$A_{t+1} - \bar{A} \approx \frac{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}{\beta\bar{\pi}_{12}}(A_t - \bar{A}) - \frac{1}{\beta}(A_{t-1} - \bar{A}).$$

And we have $A_t = A_t$. The product of the linearized-system's eigenvalues is $\frac{1}{\beta} > 1$, and the sum of the linearized system's eigenvalues is $\frac{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}{\beta\bar{\pi}_{12}}$, which is positive if and only if $\bar{\pi}_{12} > 0$.

First, assume that $\bar{\pi}_{12} > 0$. Both eigenvalues are positive and at least one is greater than 1. The characteristic equation is

$$\lambda^2 - \frac{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}{\beta\bar{\pi}_{12}}\lambda + \frac{1}{\beta},$$

where λ defines the eigenvalues. The smaller eigenvalue is less than 1 if and only if the characteristic equation is negative at $\lambda = 1$, and therefore if and only if

$$-\bar{\pi}_{11} - \beta\bar{\pi}_{22} > (1+\beta)\bar{\pi}_{12}.$$

In this case, the linearized system is saddle-path stable.

Now assume that $\bar{\pi}_{12} < 0$. Both eigenvalues are negative and at least one is less than -1. The characteristic equation is as before. The larger eigenvalue is greater than -1 if and only if the characteristic equation is negative at $\lambda = -1$, and therefore if and only if

$$\begin{aligned} \frac{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}{\bar{\pi}_{12}} + 1 + \beta &< 0 \\ \Leftrightarrow -\bar{\pi}_{11} - \beta\bar{\pi}_{22} &> -(1+\beta)\bar{\pi}_{12}. \end{aligned}$$

In this case, the linearized system is saddle-path stable.

The proposition follows from a standard application of the Hartman-Grobman theorem and from noticing that the conditions for saddle-path stability imply the condition for uniqueness.

B Proof of Lemma 2

I first describe A_t, under the assumption that $(A_{t-1}-\bar{A})^2$ is small. Write A_{t+1} as $A(A_t, w_{t+1}, f_{t+1}, w_t; \zeta)$. Expanding the stochastic Euler equation around $\zeta = 0$ and noting that all terms of order ζ^2 or larger depend on at least the third derivative of π, either Assumption 1 or 2 ensures that we can drop all terms of order ζ^2 or larger. We therefore have:

$$\begin{aligned}
0 =&\pi_1(A_t, A_{t-1}, w_t, w_{t-1}) + \beta E_t\left[\pi_2(\tilde{A}_{t+1}, A_t, f_t, w_t) + \pi_{23}(\tilde{A}_{t+1}, A_t, f_t, w_t)\epsilon_{t+1}\zeta\right] \\
&+ \beta E_t\left[\pi_{12}(\tilde{A}_{t+1}, A_t, f_t, w_t)\left(\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{\zeta=0} + \left.\frac{\partial A_{t+1}}{\partial w_{t+1}}\right|_{\zeta=0}\epsilon_{t+1} + \sum_{i=1}^{N}\left.\frac{\partial A_{t+1}}{\partial f_{i(t+1)}}\right|_{\zeta=0}\epsilon_{i(t+1)}\right)\zeta\right] \\
=&\pi_1(A_t, A_{t-1}, w_t, w_{t-1}) + \beta\pi_2(\tilde{A}_{t+1}, A_t, f_t, w_t) + \beta\pi_{12}(\tilde{A}_{t+1}, A_t, f_t, w_t)\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{\zeta=0}\zeta, \quad \text{(B-1)}
\end{aligned}$$

where $\tilde{A}_{t+1} \triangleq A(A_t, f_t, C, w_t; 0)$.

I next establish two lemmas. The first one shows that uncertainty does not have a first-order effect on policy:

Lemma 5. $\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)} = 0.$

Proof. Equation (B-1) defines A_t as a function of A_{t-1}, w_t, f_t, and ζ. Note that

$$\begin{aligned}
\left.\frac{\partial A_t}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)} =&\frac{\beta\bar{\pi}_{12}\left(\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)} + \left.\frac{\partial^2 A_{t+1}}{\partial \zeta^2}\right|_{(\bar{A},C,C,C;0)}\zeta\right)}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{122}\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{\zeta=0}\zeta - \beta\bar{\pi}_{12}\left.\frac{\partial^2 A_{t+1}}{\partial \zeta \partial A_t}\right|_{\zeta=0}\zeta} \\
=&\frac{\beta\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}\left.\frac{\partial A_{t+1}}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)},
\end{aligned}$$

where the second equality applies $\zeta = 0$. Forward-substituting, we have:

$$\left.\frac{\partial A_t}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)} = \left(\frac{\beta\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}\right)^j\left.\frac{\partial A_{t+j}}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)}$$

for $j \in \mathcal{Z}^+$. The term in parentheses is < 1 by the condition imposed following Lemma 1. Because A_{t+j} evaluated around $A_{t+j-1} = \bar{A}$, $w_t = C$, $f_t = C$, and $\zeta = 0$ must be $\bar{A}$, we know that A_{t+j} is not infinite. The derivative on the right-hand side must also be finite, in which case the right-hand side goes to 0 as j becomes large. Therefore:

$$\left.\frac{\partial A_t}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)} = 0.$$

Because the choice of t was arbitrary, we have established the lemma.

□

The second lemma solves for $\tilde{A}_{t+1}$:

Lemma 6. *If either Assumption 1 or 2 holds and* $(A_t - \bar{A})^2$ *is small, then there exists* λ *such that* $|\lambda| < 1$ *and*

$$\begin{aligned}\tilde{A}_{t+1} =& \bar{A} + \frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}(A_t - \bar{A}) + \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}(w_t - C)\\ &+ \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}(f_t - C).\end{aligned}$$

Proof. For $\zeta = 0$, the weather in period $t+1$ matches the forecast in f_t, and the weather is always C after period $t+1$. Begin by solving for policy after period $t+1$. The characteristic equation given in the proof of Lemma 1 implies the following two eigenvalues:

$$\lambda, \mu = \frac{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}{2\beta\bar{\pi}_{12}} \pm \sqrt{\left(\frac{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}{2\beta\bar{\pi}_{12}}\right)^2 - \frac{1}{\beta}}.$$

The proof of Lemma 1 showed that the two eigenvalues have the same sign. Let λ be the eigenvalue that is smallest in absolute value. We seek the eigenvector corresponding to λ, which is the stable manifold. The eigenvector is defined by $(A_{t+2} - \bar{A}) - \lambda(A_{t+1} - \bar{A}) = 0$, and thus

$$\tilde{A}_{t+2} = \lambda A_{t+1} + (1-\lambda)\bar{A},$$

for some A_{t+1}.

Now consider policy at time $t+1$. The relevant Euler equation is:

$$0 = \pi_1(\tilde{A}_{t+1}, A_t, f_t, w_t) + \beta\pi_2(\tilde{A}_{t+2}, \tilde{A}_{t+1}, C, f_t),$$

where we recognize that $w_{t+1} = f_t$. A first-order approximation to $\tilde{A}_{t+1}$ around $\bar{A}$ and the solution for $\tilde{A}_{t+2}$ is exact when either Assumption 1 or 2 holds and $(A_t - \bar{A})^2$ is small. We then have the expression in the lemma. □

Applying Lemmas 5 and 6 to equation (B-1), we have:

$$0 = \pi_1(A_t, A_{t-1}, w_t, w_{t-1}) + \beta\pi_2(\tilde{A}_{t+1}(A_t, f_t, w_t), A_t, f_t, w_t). \tag{B-2}$$

We now have A_t implicitly defined as $A(A_{t-1}, w_t, f_t, w_{t-1}; 0)$. If $(A_{t-1} - \bar{A})^2$ is small and either Assumption 1 or 2 holds, then we have:

$$\begin{aligned}
A_t =& \bar{A} + \left.\frac{\partial A_t}{\partial A_{t-1}}\right|_{(\bar{A},C,C,C;0)}(A_{t-1}-\bar{A}) + \left.\frac{\partial A_t}{\partial w_t}\right|_{(\bar{A},C,C,C;0)}(w_t - C) + \left.\frac{\partial A_t}{\partial f_t}\right|_{(\bar{A},C,C,C;0)}(f_t - C)\\
&+ \left.\frac{\partial A_t}{\partial w_{t-1}}\right|_{(\bar{A},C,C,C;0)}(w_{t-1}-C) + \left.\frac{\partial A_t}{\partial \zeta}\right|_{(\bar{A},C,C,C;0)}\zeta\\
=& \bar{A} + \frac{\bar{\pi}_{12}}{\chi}(A_{t-1}-\bar{A}) + \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}(w_t - C)\\
&+ \frac{\beta\bar{\pi}_{23} + \beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}(f_t - C) + \frac{\bar{\pi}_{14}}{\chi}(w_{t-1}-C),
\end{aligned}$$

where we use Lemma 5 in the first equality and where

$$\chi \triangleq -\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}.$$

The condition imposed following Lemma 1 and the fact that $|\lambda| < 1$ together ensure that $\chi > |\bar{\pi}_{12}|$.

Now use this result to analyze $E_0[A_t]$. If either Assumption 1 or 2 holds and $E_0[(A_1 - \bar{A})^2]$ is small, then

$$E_0[A_2] = \bar{A} + \frac{\bar{\pi}_{12}}{\chi}(E_0[A_1] - \bar{A}).$$

$E_0[(A_2 - \bar{A})^2]$ must be small because $|\bar{\pi}_{12}|/\chi < 1$. Iterating forward, we find, for $t > 1$,

$$E_0[A_t] = \bar{A} + \left(\frac{\bar{\pi}_{12}}{\chi}\right)^{t-1}(E_0[A_1] - \bar{A}).$$

As $t \to \infty$, we have:

$$E_0[A_t] \to \bar{A}.$$

We have proved the desired result.

C Proof of Proposition 1

The estimated coefficients are $\hat{\theta} = E[\tilde{X}_K^T \tilde{X}_K]^{-1} E[\tilde{X}_K \pi_t]$, where each row of $\tilde{X}_k$ is

$$\begin{bmatrix} w_{jt} - C & f_{jt} - C & w_{j(t-1)} - C & f_{j(t-1)} - C & \ldots & w_{j(t-K)} - C & f_{j(t-K)} - C \end{bmatrix}$$

and the rows correspond to the J observations. Subtracting C demeans each covariate, as implied by the fixed effects. The following lemma establishes that the coefficients on w_{jt}, f_{jt}, and $w_{j(t-1)}$ are the same for all $K \geq 2$:

Lemma 7. *It $K \geq 2$, then the first four elements of $E[\tilde{X}_K^T \tilde{X}_K]^{-1} E[\tilde{X}_K \pi_t]$ are identical to the first four elements of $E[\tilde{X}_2^T \tilde{X}_2]^{-1} E[\tilde{X}_2 \pi_t]$.*

Proof. The goal is to show that the first four rows of $E[\tilde{X}_K^T \tilde{X}_K]^{-1}$ are equal to the first four rows of $E[\tilde{X}_2^T \tilde{X}_2]^{-1}$ extended to have zeros in columns 7 through $2(K+1)$.

The case with $K = 2$ holds trivially, so assume that $K > 2$. First, note that

$$E[\tilde{X}_K^T \tilde{X}_K] = \begin{bmatrix} E[\tilde{X}_{K-1}^T \tilde{X}_{K-1}] & C_{K-1} \\ C_{K-1}^T & D \end{bmatrix},$$

where C_{K-1} is a $2K \times 2$ matrix with the only nonzero entries being in row $2K-1$, which is $[J\zeta^2\rho \quad J\zeta^2\tau^2]$, and where

$$D = J\zeta^2 \begin{bmatrix} \sigma^2 + \tau^2 & \rho \\ \rho & \tau^2 \end{bmatrix}.$$

Define

$$B_K \triangleq \begin{bmatrix} E[\tilde{X}_{K-1}^T \tilde{X}_{K-1}] & C_{K-1} \\ C_{K-1}^T & \hat{D} \end{bmatrix},$$

where

$$\hat{D} = J\zeta^2 \begin{bmatrix} \sigma^2 & \rho \\ \rho & \tau^2 \end{bmatrix}.$$

Note that[45]

$$B_{K-1} = E[\tilde{X}_{K-1}^T \tilde{X}_{K-1}] - C_{K-1} D^{-1} C_{K-1}^T.$$

Then, using standard results for block matrix inversion,

$$E[\tilde{X}_K^T \tilde{X}_K]^{-1} = \begin{bmatrix} B_{K-1}^{-1} & -B_{K-1}^{-1} C_{K-1} D^{-1} \\ -D^{-1} C_{K-1}^T B_{K-1}^{-1} & D^{-1} + D^{-1} C_{K-1}^T B_{K-1}^{-1} C_{K-1} D^{-1} \end{bmatrix}.$$

We are concerned with the top row. Note that $C_{K-1} D^{-1}$ has zeros up to row $2K-1$, so it has zeros in its first four rows. It remains to consider B_{K-1}^{-1}. We prove by induction that B_{K-1}^{-1} has the desired form. The basis step considers B_2^{-1}. This is

$$B_2^{-1} = \frac{1}{J\zeta^2[\sigma^2\tau^2 - \rho^2]} \begin{bmatrix} \tau^2 & -\rho & 0 & -\tau^2 & 0 & 0 \\ -\rho & \sigma^2 & 0 & \rho & 0 & 0 \\ 0 & 0 & \tau^2 & -\rho & 0 & -\tau^2 \\ -\tau^2 & \rho & -\rho & \sigma^2 + \tau^2 & 0 & \rho \\ 0 & 0 & 0 & 0 & \tau^2 & -\rho \\ 0 & 0 & -\tau^2 & \rho & -\rho & \sigma^2 + \tau^2 \end{bmatrix}.$$

[45]The only nonzero entries in the $2K \times 2$ matrix $C_{K-1} D^{-1}$ are in row $2K-1$, which is $[0 \quad 1]$, and thus the only nonzero entry in the $2K \times 2K$ matrix $C_{K-1} D^{-1} C_{K-1}^T$ is entry $(2K-1, 2K-1)$, which is $\zeta^2\tau^2$. Subtracting this entry from $E[\tilde{X}_{K-1}^T \tilde{X}_{K-1}]$ yields the result.

The first four rows are identical to $E[\tilde{X}_2^T \tilde{X}_2]^{-1}$. Our desired result therefore holds for the basis step. We now turn to the induction step. The induction hypothesis is that the first four rows of B_N^{-1} are equal to the first four rows of $E[\tilde{X}_2^T \tilde{X}_2]^{-1}$ extended to have zeros in columns 7 through $2(N+1)$, for some $N \geq 2$. We want to establish this result for B_{N+1}^{-1}. We have[46]

$$B_N = E[\tilde{X}_N^T \tilde{X}_N] - C_{K-1} \hat{D}^{-1} C_{K-1}^T$$

and thus

$$B_{N+1}^{-1} = \begin{bmatrix} B_N^{-1} & -B_N^{-1} C_N \hat{D}^{-1} \\ -\hat{D}^{-1} C_N^T B_N^{-1} & \hat{D}^{-1} + \hat{D}^{-1} C_N^T B_N^{-1} C_N \hat{D}^{-1} \end{bmatrix}.$$

Note that $C_N \hat{D}^{-1}$ has zeros up to row $2(N+1)-1$, so it has zeros in its first four rows. Applying the induction hypothesis, the first four rows of B_{N+1}^{-1} are equal to the first four rows of $E[\tilde{X}_2^T \tilde{X}_2]^{-1}$ extended to have zeros in columns 7 through $2(N+2)$. As a result, the first four rows of $E[\tilde{X}_K^T \tilde{X}_K]^{-1}$ are equal to the first four rows of $E[\tilde{X}_2^T \tilde{X}_2]^{-1}$ extended to have zeros in columns 7 through $2(K+1)$. We have proved the desired result.

□

It therefore suffices to analyze $K = 1$ and $K = 2$ when deriving coefficients on lags of up to one period. Note that:

$$E[\tilde{X}_1^T \tilde{X}_1] = J\zeta^2 \begin{bmatrix} \sigma^2+\tau^2 & \rho & \rho & \tau^2 \\ \rho & \tau^2 & 0 & 0 \\ \rho & 0 & \sigma^2+\tau^2 & \rho \\ \tau^2 & 0 & \rho & \tau^2 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & \tau^2 & 0 \end{bmatrix},$$

$$E[\tilde{X}_1^T \tilde{X}_1]^{-1} = \frac{1}{J\zeta^2} \begin{bmatrix} \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} \\ \frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2} \\ 0 & 0 & \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2} \\ \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2} & \frac{\tau^2(\sigma^2+\tau^2)^2-\rho^2(\sigma^2+2\tau^2)}{\rho^4+\sigma^2\tau^4(\sigma^2+\tau^2)-\rho^2(2\sigma^2\tau^2+\tau^4)} \end{bmatrix},$$

$$E[\tilde{X}_2^T \tilde{X}_2] = J\zeta^2 \begin{bmatrix} \sigma^2+\tau^2 & \rho & \rho & \tau^2 & 0 & 0 \\ \rho & \tau^2 & 0 & 0 & 0 & 0 \\ \rho & 0 & \sigma^2+\tau^2 & \rho & \rho & \tau^2 \\ \tau^2 & 0 & \rho & \tau^2 & 0 & 0 \\ 0 & 0 & \rho & 0 & \sigma^2+\tau^2 & \rho \\ 0 & 0 & \tau^2 & 0 & \rho & \tau^2 \end{bmatrix},$$

[46] The only nonzero entries in the $2K \times 2$ matrix $C_{K-1}\hat{D}^{-1}$ are in row $2K-1$, which is $[0 \quad 1]$, and thus the only nonzero entry in the $2K \times 2K$ matrix $C_{K-1}\hat{D}^{-1}C_{K-1}^T$ is entry $(2K-1, 2K-1)$, which is $\zeta^2\tau^2$. Subtracting this entry from $E[\tilde{X}_N^T \tilde{X}_N]$ yields the result.

and

$$E[\tilde{X}_2^T\tilde{X}_2]^{-1} = \frac{1}{J\zeta^2}\begin{bmatrix} \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & 0 \\ \frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2} & 0 & 0 \\ 0 & 0 & \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} \\ \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2+\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2} \\ 0 & 0 & 0 & 0 & \frac{\tau^2}{\tau^2(\sigma^2+\tau^2)-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2} \\ 0 & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2} & \frac{-\tau^2(\sigma^2+\tau^2)^2+\rho^2(\sigma^2+2\tau^2)}{(\rho^2-\sigma^2\tau^2)(\tau^2(\sigma^2+\tau^2)-\rho^2)} \end{bmatrix}.$$

We also have:

$$E[\tilde{X}_1^T\pi_{jt}] = J\begin{bmatrix} Cov[w_{jt}-C,\pi_{jt}] \\ Cov[f_{jt}-C,\pi_{jt}] \\ Cov[w_{j(t-1)}-C,\pi_{jt}] \\ Cov[f_{j(t-1)}-C,\pi_{jt}] \end{bmatrix},$$

$$E[\tilde{X}_2^T\pi_{jt}] = J\begin{bmatrix} Cov[w_{jt}-C,\pi_{jt}] \\ Cov[f_{jt}-C,\pi_{jt}] \\ Cov[w_{j(t-1)}-C,\pi_{jt}] \\ Cov[f_{j(t-1)}-C,\pi_{jt}] \\ Cov[w_{j(t-2)}-C,\pi_{jt}] \\ Cov[f_{j(t-2)}-C,\pi_{jt}] \end{bmatrix}.$$

From here, drop the j subscript to save on unnecessary notation. Consider $Cov[w_t - C, \pi_t]$. Expanding π around $A_t = \bar{A}$, $A_{t-1} = \bar{A}$, $w_t = C$, and $w_{t-1} = C$, applying either Assumption 1 or 2, and assuming that $(A_t - \bar{A})^2$ and $(A_{t-1} - \bar{A})^2$ are small, we have:

$$\begin{aligned} \pi(A_t, A_{t-1}, w_t, w_{t-1}) =& \bar{\pi} + \bar{\pi}_1(A_t - \bar{A}) + \bar{\pi}_2(A_{t-1} - \bar{A}) + \bar{\pi}_3(w_t - C) + \bar{\pi}_4(w_{t-1} - C) \\ &+ \frac{1}{2}\bar{\pi}_{33}(w_t - C)^2 + \bar{\pi}_{13}(A_t - \bar{A})(w_t - C) + \bar{\pi}_{23}(A_{t-1} - \bar{A})(w_t - C) \\ &+ \frac{1}{2}\bar{\pi}_{44}(w_{t-1} - C)^2 + \bar{\pi}_{14}(A_t - \bar{A})(w_{t-1} - C) + \bar{\pi}_{24}(A_{t-1} - \bar{A})(w_{t-1} - C) \\ &+ \bar{\pi}_{34}(w_t - C)(w_{t-1} - C). \end{aligned}$$

As a result,

$$\begin{aligned}
Cov[w_t - C, \pi_t] =&\bar{\pi}_1 Cov[A_t, w_t] + \bar{\pi}_2 Cov[A_{t-1}, w_t] + \bar{\pi}_3 Var[w_t] + \bar{\pi}_4 Cov[w_t, w_{t-1}]\\
&+ \frac{1}{2}\bar{\pi}_{33} Cov[w_t - C, (w_t - C)^2] + \frac{1}{2}\bar{\pi}_{44} Cov[w_t - C, (w_{t-1} - C)^2]\\
&- C\bar{\pi}_{13} Cov[A_t, w_t] - \bar{A}\bar{\pi}_{13} Var[w_t] + \bar{\pi}_{13} Cov[w_t, A_t w_t]\\
&- C\bar{\pi}_{23} Cov[A_{t-1}, w_t] - \bar{A}\bar{\pi}_{23} Var[w_t] + \bar{\pi}_{23} Cov[w_t, A_{t-1} w_t]\\
&- C\bar{\pi}_{14} Cov[w_t, A_t] - \bar{A}\bar{\pi}_{14} Cov[w_t, w_{t-1}] + \bar{\pi}_{14} Cov[w_t, A_t w_{t-1}]\\
&- C\bar{\pi}_{24} Cov[w_t, A_{t-1}] - \bar{A}\bar{\pi}_{24} Cov[w_t, w_{t-1}] + \bar{\pi}_{24} Cov[w_t, A_{t-1} w_{t-1}]\\
&- C\bar{\pi}_{34} Var[w_t] - C\bar{\pi}_{34} Cov[w_t, w_{t-1}] + \bar{\pi}_{34} Cov[w_t, w_t w_{t-1}].
\end{aligned}$$

If the ϵ and ν are normally distributed, then $Cov[w_t - C, (w_t - C)^2] = 0$, or if Assumption 1 holds, then $Cov[w_t - C, (w_t - C)^2] \approx 0$. Using results from Bohrnstedt and Goldberger (1969), we have:

$$Cov[w_t - C, (w_{t-1} - C)^2] = E[(w_t - C)(w_{t-1} - C)^2],$$

which is zero if either the ϵ and ν are normally distributed or Assumption 1 holds. Again using results from Bohrnstedt and Goldberger (1969), we also have:

$$Cov[w_t, A_t w_t] = E[A_t]\, Var[w_t] + C\, Cov[A_t, w_t] + E[(w_t - C)^2 (A_t - E[A_t])].$$

If either the ϵ and ν are normally distributed or Assumption 1 holds, then this becomes :

$$Cov[w_t, A_t w_t] = E[A_t]\, Var[w_t] + C\, Cov[A_t, w_t].$$

Analogous derivations yield:

$$\begin{aligned}
Cov[w_t, A_{t-1} w_t] =& E[A_{t-1}]\, Var[w_t] + C\, Cov[w_t, A_{t-1}],\\
Cov[w_t, A_t w_{t-1}] =& E[A_t]\, Cov[w_t, w_{t-1}] + C\, Cov[w_t, A_t],\\
Cov[w_t, A_{t-1} w_{t-1}] =& E[A_{t-1}]\, Cov[w_t, w_{t-1}] + C\, Cov[w_t, A_{t-1}]
\end{aligned}$$

if either the ϵ and ν are normally distributed or Assumption 1 holds. Substituting these results in, we find:

$$\begin{aligned}
Cov[w_t - C, \pi_t] =&\bar{\pi}_1 Cov[A_t, w_t] + \bar{\pi}_2 Cov[A_{t-1}, w_t] + \bar{\pi}_3 Var[w_t] + \bar{\pi}_4 Cov[w_t, w_{t-1}]\\
&+ \left(E[A_t] - \bar{A}\right)\left(\bar{\pi}_{13} Var[w_t] + \bar{\pi}_{14} Cov[w_t, w_{t-1}]\right)\\
&+ \left(E[A_{t-1}] - \bar{A}\right)\left(\bar{\pi}_{23} Var[w_t] + \bar{\pi}_{24} Cov[w_t, w_{t-1}]\right).
\end{aligned}$$

The assumption that actions are on average around $\bar{A}$ implies $E[A_t] = \bar{A}$ and $E[A_{t-1}] = \bar{A}$. Using that and equation (7), we obtain:

$$\frac{1}{\zeta^2}Cov[w_t - C, \pi_t] =(\sigma^2+\tau^2)\bar{\pi}_3 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left((\sigma^2+\tau^2)\bar{\pi}_1 + \bar{\pi}_2\rho + \bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\rho\right)$$
$$+ \bar{\pi}_4\rho + \frac{\bar{\pi}_{14}}{\chi}\bar{\pi}_1\rho + \frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_1\rho + \bar{\pi}_2\tau^2 + \bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\tau^2\right).$$

If Assumption 3 holds, then the Euler equation requires $\bar{\pi}_2 = \bar{\pi}_1 = 0$. We therefore have:

$$\frac{1}{\zeta^2}Cov[w_t - C, \pi_t] =(\sigma^2+\tau^2)\bar{\pi}_3 + \bar{\pi}_4\rho.$$

Analogous derivations yield:

$$\frac{1}{\zeta^2}Cov[f_t - C, \pi_t] =\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t, f_t] + \bar{\pi}_3 Cov[w_t, f_t]\right)$$
$$=\bar{\pi}_3\rho,$$

$$\frac{1}{\zeta^2}Cov[w_{t-1} - C, \pi_t] =\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t, w_{t-1}] + \bar{\pi}_2 Cov[A_{t-1}, w_{t-1}] + \bar{\pi}_3 Cov[w_t, w_{t-1}] + \bar{\pi}_4 Var[w_{t-1}]\right)$$
$$=\bar{\pi}_3\rho + \bar{\pi}_4(\sigma^2+\tau^2),$$

$$\frac{1}{\zeta^2}Cov[f_{t-1} - C, \pi_t] =\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t, f_{t-1}] + \bar{\pi}_2 Cov[A_{t-1}, f_{t-1}] + \bar{\pi}_3 Cov[w_t, f_{t-1}] + \bar{\pi}_4 Cov[w_{t-1}, f_{t-1}]\right)$$
$$=\bar{\pi}_3\tau^2 + \bar{\pi}_4\rho,$$

$$\frac{1}{\zeta^2}Cov[w_{t-2} - C, \pi_t] =\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t, w_{t-2}] + \bar{\pi}_2 Cov[A_{t-1}, w_{t-2}] + \bar{\pi}_3 Cov[w_t, w_{t-2}] + \bar{\pi}_4 Cov[w_{t-1}, w_{t-2}]\right)$$
$$=\bar{\pi}_4\rho,$$

$$\frac{1}{\zeta^2}Cov[f_{t-2} - C, \pi_t] =\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t, f_{t-2}] + \bar{\pi}_2 Cov[A_{t-1}, f_{t-2}] + \bar{\pi}_3 Cov[w_t, f_{t-2}] + \bar{\pi}_4 Cov[w_{t-1}, f_{t-2}]\right)$$
$$=\bar{\pi}_4\tau^2.$$

For either $K=1$ or $K=2$, simple calculations show that $\hat{\theta}_{w_t} = \bar{\pi}_3$, $\hat{\theta}_{w_{t-1}} = \bar{\pi}_4$, and $\hat{\theta}_{f_t} = 0$. The proposition follows.

D Proof of Lemma 4

Using equation (7) and standard regression properties, we have:

$$\hat{\Gamma}_1 = \omega \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}},$$
$$\hat{\Gamma}_2 = \omega \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}},$$
$$\hat{\Gamma}_3 = \omega \frac{\beta\bar{\pi}_{23} + \beta(\bar{\pi}_{13} + \beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}},$$
$$\hat{\Gamma}_4 = \omega \frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}},$$

where

$$\omega \triangleq \frac{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}}{\chi} > 0.$$

Note that $\omega > 1$ if $\bar{\pi}_{12} < 0$, $\omega = 1$ if $\bar{\pi}_{12} = 0$, and $\omega < 1$ if $\bar{\pi}_{12} > 0$. The lemma follows from defining

$$\Omega \triangleq \frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}.$$

The sign of Ω matches the sign of $\bar{\pi}_{12}$.

E Proof of Proposition 2

The proof of Lemma 4 provides expressions for $\hat{\Gamma}_1$, $\hat{\Gamma}_2$, $\hat{\Gamma}_3$, and $\hat{\Gamma}_4$. Now consider the parts of the proposition:

1. Follows directly from the expressions in the proof of Lemma 4.
2. If $\bar{\pi}_{12} = 0$, then $\omega = 1$. The result follows from Lemma 4.
3. If $\bar{\pi}_{23}, \bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\bar{\pi}_{13} \neq 0$, then $\hat{\Gamma}_1/[\mathrm{d}\bar{A}/\mathrm{d}C] = \omega$. The result follows.
4. If $\bar{\pi}_{13}, \bar{\pi}_{14}, \bar{\pi}_{24} = 0$ and $\beta\bar{\pi}_{23} \neq 0$, then $\hat{\Gamma}_3/[\mathrm{d}\bar{A}/\mathrm{d}C] = \omega$. The result follows.
5. If $\beta = 0$ and either $\bar{\pi}_{13} \neq 0$ or $\bar{\pi}_{14} \neq 0$, then $[\hat{\Gamma}_1 + \hat{\Gamma}_2]/[\mathrm{d}\bar{A}/\mathrm{d}C] = \omega$. The result follows.

F Proof of Proposition 3

Begin by considering $\hat{\gamma}_1$. We apply the Frisch-Waugh theorem. The residuals from regressing w_{jt} on $w_{j(t-1)}$ are:

$$\tilde{w}_{jt} \triangleq w_{jt} - C - \frac{\rho}{\tau^2+\sigma^2}(w_{j(t-1)} - C) = \zeta\epsilon_{jt} + \zeta\nu_{j(t-1)} - \zeta\frac{\rho}{\tau^2+\sigma^2}[\epsilon_{j(t-1)} + \nu_{j(t-2)}].$$

We then have:

$$\begin{aligned}
\hat{\gamma}_1 =& \frac{Cov[\tilde{w}_{jt}, A_{jt}]}{Var[\tilde{w}_{jt}]} \\
=& \omega\Bigg\{ \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \\
&+ \frac{\beta\bar{\pi}_{23} + \beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \frac{\rho + \frac{\bar{\pi}_{12}}{\chi}\left(\tau^2 - \frac{\rho^2}{\tau^2+\sigma^2}\right)}{\sigma^2 + \tau^2 - \frac{\rho^2}{\tau^2+\sigma^2}} \\
&- \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \frac{\bar{\pi}_{12}}{\chi} \frac{\frac{\rho^2}{\tau^2+\sigma^2}}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}} \Bigg\}.
\end{aligned}$$

Now consider $\hat{\gamma}_2$. The residuals from regressing $w_{j(t-1)}$ on w_{jt} are:

$$\tilde{w}_{j(t-1)} \triangleq w_{j(t-1)} - C - \frac{\rho}{\tau^2+\sigma^2}(w_{jt} - C) = \zeta\epsilon_{j(t-1)} + \zeta\nu_{j(t-2)} - \zeta\frac{\rho}{\tau^2+\sigma^2}[\epsilon_{jt} + \nu_{j(t-1)}].$$

We then have:

$$\begin{aligned}
\hat{\gamma}_2 =& \omega\Bigg\{ \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \frac{\bar{\pi}_{12}}{\chi} \\
&+ \frac{\beta\bar{\pi}_{23} + \beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \frac{\frac{\bar{\pi}_{12}}{\chi}\frac{\sigma^2}{\tau^2+\sigma^2}\rho - \frac{\rho^2}{\tau^2+\sigma^2}}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}} \\
&+ \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \left(1 + \frac{\bar{\pi}_{12}}{\chi} \frac{\rho}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}}\right) \Bigg\}.
\end{aligned}$$

Now consider the parts of the proposition:

1. If $\bar{\pi}_{12} = 0$, then

$$\hat{\gamma}_1 = \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{23}\frac{\rho}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22}}.$$

Because correlation coefficients are bounded above by 1,

$$\rho \leq \sigma\tau \leq \max\{\sigma^2, \tau^2\}$$

and

$$\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2} \geq \sigma^2+\tau^2-\frac{\tau^2\sigma^2}{\tau^2+\sigma^2} = \max\{\sigma^2,\tau^2\}+\min\{\sigma^2,\tau^2\}\left[1-\frac{\max\{\sigma^2,\tau^2\}}{\tau^2+\sigma^2}\right].$$

Therefore

$$\frac{\rho}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}} \leq \frac{\max\{\sigma^2,\tau^2\}}{\max\{\sigma^2,\tau^2\}+\min\{\sigma^2,\tau^2\}\left[1-\frac{\max\{\sigma^2,\tau^2\}}{\tau^2+\sigma^2}\right]} < 1. \tag{F-3}$$

The results follow.

2. If $\bar{\pi}_{12}=0$, then

$$\hat{\gamma}_2 = \frac{\bar{\pi}_{14}-\beta\bar{\pi}_{23}\frac{\frac{\rho^2}{\tau^2+\sigma^2}}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}}}{-\bar{\pi}_{11}-(1+\beta)\bar{\pi}_{12}-\beta\bar{\pi}_{22}}$$

and

$$\hat{\gamma}_1+\hat{\gamma}_2 = \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}+\beta\bar{\pi}_{23}\frac{\rho\left(1-\frac{\rho}{\tau^2+\sigma^2}\right)}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}}}{-\bar{\pi}_{11}-(1+\beta)\bar{\pi}_{12}-\beta\bar{\pi}_{22}}.$$

From equation (F-3),

$$\hat{\gamma}_1+\hat{\gamma}_2 \leq \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}+\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-(1+\beta)\bar{\pi}_{12}-\beta\bar{\pi}_{22}},$$

with strict inequality if $\beta\bar{\pi}_{23}>0$ and equality if $\beta\bar{\pi}_{23}=0$. Because

$$1-\frac{\rho}{\tau^2+\sigma^2} \geq 1-\frac{\sigma\tau}{\tau^2+\sigma^2} > 0,$$

we have

$$\hat{\gamma}_1+\hat{\gamma}_2 > \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}}{-\bar{\pi}_{11}-(1+\beta)\bar{\pi}_{12}-\beta\bar{\pi}_{22}}$$

if $\rho\beta\bar{\pi}_{23}>0$. Finally, note that, for $\bar{\pi}_{12}=0$,

$$\hat{\Gamma}_1+\hat{\Gamma}_2 = \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}}{-\bar{\pi}_{11}-(1+\beta)\bar{\pi}_{12}-\beta\bar{\pi}_{22}}.$$

The results follow.

3. If $\beta=0$, then

$$\hat{\gamma}_1 = \omega\left\{\frac{\bar{\pi}_{13}}{-\bar{\pi}_{11}-\bar{\pi}_{12}} - \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\bar{\pi}_{12}}\frac{\bar{\pi}_{12}}{\chi}\frac{\frac{\rho^2}{\tau^2+\sigma^2}}{\sigma^2+\tau^2-\frac{\rho^2}{\tau^2+\sigma^2}}\right\}.$$

$\omega > 1$ if and only if $\bar{\pi}_{12} < 0$. If $\bar{\pi}_{12} > 0$ then

$$\hat{\gamma}_1 < \frac{\bar{\pi}_{13}}{-\bar{\pi}_{11} - \bar{\pi}_{12}} \leq \frac{\mathrm{d}\bar{A}}{\mathrm{d}C}.$$

If $\bar{\pi}_{12} < 0$ and $\bar{\pi}_{14} = 0$, then

$$\hat{\gamma}_1 > \frac{\bar{\pi}_{13}}{-\bar{\pi}_{11} - \bar{\pi}_{12}} = \frac{\mathrm{d}\bar{A}}{\mathrm{d}C}.$$

We have established the result.

G Proof of Proposition 4

The estimated coefficients are $\hat{\theta} = E[\tilde{X}_K^T \tilde{X}_K]^{-1} E[\tilde{X}_K \pi_t]$, where each row of $\tilde{X}_k$ is

$$\begin{bmatrix} w_{jt} - C & f_{jt} - C & w_{j(t-1)} - C & f_{j(t-1)} - C & \dots & w_{j(t-K)} - C & f_{j(t-K)} - C \end{bmatrix}$$

and the rows correspond to the J observations. Subtracting C demeans each covariate, as implied by the fixed effects. Following the steps in Lemma 7, it is easy to show that we need only analyze a case with $K = 3$ in order to derive the coefficients on each $w_{j(t-n)}$ and $f_{j(t-n)}$ for $n \in \{0, 1, 2\}$.

We therefore focus on $K = 3$ when deriving coefficients on lags of up to two periods. Note that:

$$E[\tilde{X}_3^T \tilde{X}_3] = J\zeta^2 \begin{bmatrix} \sigma^2 + \tau^2 & \rho & \rho & \tau^2 & 0 & 0 & 0 & 0 \\ \rho & \tau^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho & 0 & \sigma^2 + \tau^2 & \rho & \rho & \tau^2 & 0 & 0 \\ \tau^2 & 0 & \rho & \tau^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & \sigma^2 + \tau^2 & \rho & \rho & \tau^2 \\ 0 & 0 & \tau^2 & 0 & \rho & \tau^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho & 0 & \sigma^2 + \tau^2 & \rho \\ 0 & 0 & 0 & 0 & \tau^2 & 0 & \rho & \tau^2 \end{bmatrix}$$

and

$$
\begin{aligned}
&E[\tilde{X}_3^T\tilde{X}_3]^{-1}\\
=&\frac{1}{J\zeta^2}\begin{bmatrix}
\frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & 0 & 0 & 0\\
\frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2} & 0 & 0 & 0 & 0\\
0 & 0 & \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & 0\\
\frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2+\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2} & 0 & 0\\
0 & 0 & 0 & 0 & \frac{\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2}\\
0 & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\sigma^2\tau^2-\rho^2} & \frac{\sigma^2+\tau^2}{\sigma^2\tau^2-\rho^2} & 0 & \frac{\rho}{\sigma^2\tau^2-\rho^2}\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\tau^2}{\tau^2(\sigma^2+\tau^2)-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2}\\
0 & 0 & 0 & 0 & \frac{-\tau^2}{\sigma^2\tau^2-\rho^2} & \frac{\rho}{\sigma^2\tau^2-\rho^2} & \frac{-\rho}{\tau^2(\sigma^2+\tau^2)-\rho^2} & \frac{\tau^2(\sigma^2+\tau^2)^2-\rho^2(\sigma^2+2\tau^2)}{\rho^4+\sigma^2\tau^4(\sigma^2+\tau^2)-\rho^2\tau^2(2\sigma^2+\tau^2)}
\end{bmatrix}.
\end{aligned}
$$

We also have:

$$
E[\tilde{X}_3^T\pi_{jt}] = J\begin{bmatrix}
Cov[w_{jt}-C,\pi_{jt}]\\
Cov[f_{jt}-C,\pi_{jt}]\\
Cov[w_{j(t-1)}-C,\pi_{jt}]\\
Cov[f_{j(t-1)}-C,\pi_{jt}]\\
Cov[w_{j(t-2)}-C,\pi_{jt}]\\
Cov[f_{j(t-2)}-C,\pi_{jt}]\\
Cov[w_{j(t-3)}-C,\pi_{jt}]\\
Cov[f_{j(t-3)}-C,\pi_{jt}]
\end{bmatrix}.
$$

From here, drop the j subscript to save on unnecessary notation. Following the proof of Proposition 1, we have:

$$
\begin{aligned}
\frac{1}{\zeta^2}Cov[w_t-C,\pi_t] =& (\sigma^2+\tau^2)\bar{\pi}_3 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left((\sigma^2+\tau^2)\bar{\pi}_1+\bar{\pi}_2\rho+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\rho\right)\\
&+\bar{\pi}_4\rho+\frac{\bar{\pi}_{14}}{\chi}\bar{\pi}_1\rho+\frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_1\rho+\bar{\pi}_2\tau^2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\tau^2\right).
\end{aligned}
$$

Analogous derivations yield:

$$
\begin{aligned}
\frac{1}{\zeta^2}Cov[f_t-C,\pi_t] =& \frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t,f_t]+\bar{\pi}_3 Cov[w_t,f_t]\right)\\
=& \bar{\pi}_3\rho+\frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\bar{\pi}_1\rho\\
&+\frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\bar{\pi}_1\tau^2,
\end{aligned}
$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[w_{t-1}-C,\pi_t] =& \frac{1}{\zeta^2}\bigg(\bar{\pi}_1 Cov[A_t,w_{t-1}] + \bar{\pi}_2 Cov[A_{t-1},w_{t-1}] + \bar{\pi}_3 Cov[w_t,w_{t-1}] + \bar{\pi}_4 Var[w_{t-1}]\bigg)\\
=& \bar{\pi}_3\rho + \bar{\pi}_4(\sigma^2+\tau^2) + \bar{\pi}_1\frac{\bar{\pi}_{14}}{\chi}(\sigma^2+\tau^2) + \left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{14}}{\chi}\rho\\
&+ \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left[\bar{\pi}_1\rho + \left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left(\sigma^2+\tau^2+\frac{\bar{\pi}_{12}}{\chi}\rho\right)\right]\\
&+ \left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left(\rho+\frac{\bar{\pi}_{12}}{\chi}\tau^2\right)\frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi},
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[f_{t-1}-C,\pi_t] =& \frac{1}{\zeta^2}\bigg(\bar{\pi}_1 Cov[A_t,f_{t-1}] + \bar{\pi}_2 Cov[A_{t-1},f_{t-1}] + \bar{\pi}_3 Cov[w_t,f_{t-1}] + \bar{\pi}_4 Cov[w_{t-1},f_{t-1}]\bigg)\\
=& \bar{\pi}_3\tau^2 + \bar{\pi}_4\rho + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_1\tau^2+\bar{\pi}_2\rho+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\rho\right)\\
&+ \frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2\tau^2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\tau^2\right)\\
&+ \bar{\pi}_1\frac{\bar{\pi}_{14}}{\chi}\rho,
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[w_{t-2}-C,\pi_t] =& \frac{1}{\zeta^2}\bigg(\bar{\pi}_1 Cov[A_t,w_{t-2}] + \bar{\pi}_2 Cov[A_{t-1},w_{t-2}] + \bar{\pi}_3 Cov[w_t,w_{t-2}] + \bar{\pi}_4 Cov[w_{t-1},w_{t-2}]\bigg)\\
=& \bar{\pi}_4\rho + \bar{\pi}_1\frac{\bar{\pi}_{14}}{\chi}\rho + \frac{\bar{\pi}_{14}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left[(\sigma^2+\tau^2)+\rho\frac{\bar{\pi}_{12}}{\chi}\right]\\
&+ \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left[\rho+\frac{\bar{\pi}_{12}}{\chi}(\sigma^2+\tau^2)+\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\rho\right]\\
&+ \frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\frac{\bar{\pi}_{12}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left[\rho+\frac{\bar{\pi}_{12}}{\chi}\tau^2\right],
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[f_{t-2}-C,\pi_t] =& \frac{1}{\zeta^2}\bigg(\bar{\pi}_1 Cov[A_t,f_{t-2}] + \bar{\pi}_2 Cov[A_{t-1},f_{t-2}] + \bar{\pi}_3 Cov[w_t,f_{t-2}] + \bar{\pi}_4 Cov[w_{t-1},f_{t-2}]\bigg)\\
=& \bar{\pi}_4\tau^2 + \bar{\pi}_1\frac{\bar{\pi}_{14}}{\chi}\tau^2 + \frac{\bar{\pi}_{14}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\rho\\
&+ \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left[\tau^2+\frac{\bar{\pi}_{12}}{\chi}\rho\right]\\
&+ \frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\tau^2,
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[w_{t-3}-C,\pi_t] =&\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t,w_{t-3}]+\bar{\pi}_2 Cov[A_{t-1},w_{t-3}]+\bar{\pi}_3 Cov[w_t,w_{t-3}]+\bar{\pi}_4 Cov[w_{t-1},w_{t-3}]\right)\\
=&\frac{\bar{\pi}_{14}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left\{\rho+\frac{\bar{\pi}_{12}}{\chi}\left[(\sigma^2+\tau^2)+\rho\frac{\bar{\pi}_{12}}{\chi}\right]\right\}\\
&+\frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\\
&\left[\rho+\frac{\bar{\pi}_{12}}{\chi}(\sigma^2+\tau^2)+\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\rho\right]\\
&+\frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left[\rho+\frac{\bar{\pi}_{12}}{\chi}\tau^2\right],
\end{aligned}$$

$$\begin{aligned}
\frac{1}{\zeta^2}Cov[f_{t-3}-C,\pi_t] =&\frac{1}{\zeta^2}\left(\bar{\pi}_1 Cov[A_t,f_{t-3}]+\bar{\pi}_2 Cov[A_{t-1},f_{t-3}]+\bar{\pi}_3 Cov[w_t,f_{t-3}]+\bar{\pi}_4 Cov[w_{t-1},f_{t-3}]\right)\\
=&\frac{\bar{\pi}_{14}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left\{\tau^2+\frac{\bar{\pi}_{12}}{\chi}\rho\right\}\\
&+\frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\left[\tau^2+\frac{\bar{\pi}_{12}}{\chi}\rho\right]\\
&+\frac{\beta\bar{\pi}_{23}+\beta(\bar{\pi}_{13}+\beta\bar{\pi}_{24})\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}-\beta\bar{\pi}_{12}\lambda}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\tau^2.
\end{aligned}$$

Now consider the regression coefficients, from $E[\tilde{X}_3^T\tilde{X}_3]^{-1}E[\tilde{X}_3^T\pi_t]$. The coefficient on w_{jt} is:

$$\begin{aligned}
\hat{\theta}_{w_t} =&\frac{1}{\zeta^2}\left(\frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[w_t-C,\pi_t]+\frac{-\rho}{\sigma^2\tau^2-\rho^2}Cov[f_t-C,\pi_t]+\frac{-\tau^2}{\sigma^2\tau^2-\rho^2}Cov[f_{t-1}-C,\pi_t]\right)\\
=&\bar{\pi}_3+\hat{\Gamma}_1\bar{\pi}_1.
\end{aligned}$$

The coefficient on f_{jt} is:

$$\begin{aligned}
\hat{\theta}_{f_t} =&\frac{1}{\zeta^2}\left(\frac{-\rho}{\sigma^2\tau^2-\rho^2}Cov[w_t-C,\pi_t]+\frac{\sigma^2}{\sigma^2\tau^2-\rho^2}Cov[f_t-C,\pi_t]+\frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[f_{t-1}-C,\pi_t]\right)\\
=&\hat{\Gamma}_3\bar{\pi}_1.
\end{aligned}$$

The coefficient on $w_{j(t-1)}$ is:

$$\begin{aligned}
\hat{\theta}_{w_{t-1}} =&\frac{1}{\zeta^2}\left(\frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[w_{t-1}-C,\pi_t]-\frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[f_{t-1}-C,\pi_t]-\frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[f_{t-2}-C,\pi_t]\right)\\
=&\bar{\pi}_4+\hat{\Gamma}_1\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2+\hat{\Gamma}_2\bar{\pi}_1.
\end{aligned}$$

The coefficient on $f_{j(t-1)}$ is:

$$\begin{aligned}\hat{\theta}_{f_{t-1}} =&\frac{1}{\zeta^2}\Bigg(- \frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[w_t - C, \pi_t] - \frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[w_{t-1} - C, \pi_t] + \frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[f_t - C, \pi_t]\\ &+ \frac{\sigma^2+\tau^2}{\sigma^2\tau^2-\rho^2}Cov[f_{t-1} - C, \pi_t] + \frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[f_{t-2} - C, \pi_t]\Bigg)\\ =&\hat{\Gamma}_3\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2.\end{aligned}$$

And the coefficient on $w_{j(t-2)}$ is:

$$\begin{aligned}\hat{\theta}_{w_{t-2}} =&\frac{1}{\zeta^2}\left(\frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[w_{t-2} - C, \pi_t] - \frac{\rho}{\sigma^2\tau^2-\rho^2}Cov[f_{t-2} - C, \pi_t] - \frac{\tau^2}{\sigma^2\tau^2-\rho^2}Cov[f_{t-3} - C, \pi_t]\right)\\ =&\hat{\Gamma}_1\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\bar{\pi}_2 + \hat{\Gamma}_2\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2.\end{aligned}$$

We now prove the parts of the proposition.

1. Directly follows from the foregoing.

2. By Proposition 2, $\bar{\pi}_{12}=0$ implies $\mathrm{d}\bar{A}/\,\mathrm{d}C = \hat{\Gamma}_1+\hat{\Gamma}_2+\hat{\Gamma}_3$. The result follows from the foregoing and equation (4).

3. Under the given assumptions, we have:

$$\hat{\theta}_{w_t}+\hat{\theta}_{w_{t-1}} =\bar{\pi}_3+\bar{\pi}_4+\hat{\Gamma}_1\bar{\pi}_1+\hat{\Gamma}_1\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2,$$

   so

$$\hat{\theta}_{w_t}+\hat{\theta}_{w_{t-1}} \leq\bar{\pi}_3+\bar{\pi}_4+\hat{\Gamma}_1\bar{\pi}_1+\hat{\Gamma}_1\bar{\pi}_2 \quad\Leftrightarrow\quad \bar{\pi}_2\bar{\pi}_{12}>0.$$

   Using Proposition 2 and equations (2) and (4),

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} >\bar{\pi}_3+\bar{\pi}_4+\hat{\Gamma}_1\bar{\pi}_1+\hat{\Gamma}_1\bar{\pi}_2 \quad\Leftrightarrow\quad \bar{\pi}_2\bar{\pi}_{12}>0,$$

   using $\bar{\pi}_{13}>0$. Therefore

$$\hat{\theta}_{w_t}+\hat{\theta}_{w_{t-1}} <\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} \quad\Leftrightarrow\quad \bar{\pi}_2\bar{\pi}_{12}>0.$$

   We have established the result.

4. Under the given assumptions, we have:

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}} = \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_3 \bar{\pi}_1 + \hat{\Gamma}_3 \left(1 - \beta \frac{\bar{\pi}_{12}}{\chi}\right) \bar{\pi}_2,$$

so

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}} < \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_3 \bar{\pi}_1 + \hat{\Gamma}_3 \bar{\pi}_2 \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0,$$

using $\beta\bar{\pi}_{23} > 0$. Using Proposition 2 and equations (2) and (4),

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} > \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_3 \bar{\pi}_1 + \hat{\Gamma}_3 \bar{\pi}_2 \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0,$$

using $\beta\bar{\pi}_{23} > 0$. Therefore

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{f_t} + \hat{\theta}_{f_{t-1}} < \frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0.$$

We have established the result.

5. It is clear that $\beta = 0$ implies $\hat{\theta}_{f_t} = 0$ and $\hat{\theta}_{f_{t-1}} = 0$. Further, the Euler equation implies $\bar{\pi}_1 = 0$. We then have:

$$\begin{aligned} \hat{\theta}_{w_t} &= \bar{\pi}_3, \\ \hat{\theta}_{w_{t-1}} &= \bar{\pi}_4 + \hat{\Gamma}_1 \bar{\pi}_2, \\ \hat{\theta}_{w_{t-2}} &= \hat{\Gamma}_2 \bar{\pi}_2 + \hat{\Gamma}_1 \frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}} \bar{\pi}_2. \end{aligned}$$

Therefore:

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} = \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_1 \left(1 + \frac{\bar{\pi}_{12}}{-\bar{\pi}_{11}}\right) \bar{\pi}_2 + \hat{\Gamma}_2 \bar{\pi}_2.$$

We then have:

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} \leq \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_1 \bar{\pi}_2 + \hat{\Gamma}_2 \bar{\pi}_2 \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0.$$

Using Proposition 2 and equations (2) and (4),

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} > \bar{\pi}_3 + \bar{\pi}_4 + \hat{\Gamma}_1 \bar{\pi}_2 + \hat{\Gamma}_2 \bar{\pi}_2 \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0,$$

using either $\bar{\pi}_{13} > 0$ or $\bar{\pi}_{14} > 0$. Therefore

$$\hat{\theta}_{w_t} + \hat{\theta}_{w_{t-1}} + \hat{\theta}_{w_{t-2}} < \frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} \quad \Leftrightarrow \quad \bar{\pi}_2 \bar{\pi}_{12} > 0.$$

We have established the result.

H Proof of Corollary 5

We follow the proof of Proposition 2 but now do not impose the assumption that $E[A_t] = \bar{A}$ and $E[A_{t-1}] = \bar{A}$. We now have:

$$\begin{aligned} Cov[w_t - C, \pi_t] =& \bar{\pi}_1 Cov[A_t, w_t] + \bar{\pi}_2 Cov[A_{t-1}, w_t] \\ &+ \left(\bar{\pi}_3 + \bar{\pi}_{13}(E[A_t] - \bar{A}) + \bar{\pi}_{23}(E[A_{t-1}] - \bar{A})\right) Var[w_t] \\ &+ \left(\bar{\pi}_4 + \bar{\pi}_{14}(E[A_t] - \bar{A}) + \bar{\pi}_{24}(E[A_{t-1}] - \bar{A})\right) Cov[w_t, w_{t-1}], \end{aligned}$$

$$Cov[f_t - C, \pi_t] = \bar{\pi}_1 Cov[A_t, f_t] + \left(\bar{\pi}_3 + \bar{\pi}_{13}(E[A_t] - \bar{A}) + \bar{\pi}_{23}(E[A_{t-1}] - \bar{A})\right) Cov[w_t, f_t],$$

$$\begin{aligned} Cov[w_{t-1} - C, \pi_t] =& \bar{\pi}_1 Cov[A_t, w_{t-1}] + \bar{\pi}_2 Cov[A_{t-1}, w_{t-1}] \\ &+ \left(\bar{\pi}_3 + \bar{\pi}_{13}(E[A_t] - \bar{A}) + \bar{\pi}_{23}(E[A_{t-1}] - \bar{A})\right) Cov[w_t, w_{t-1}] \\ &+ \left(\bar{\pi}_4 + \bar{\pi}_{14}(E[A_t] - \bar{A}) + \bar{\pi}_{24}(E[A_{t-1}] - \bar{A})\right) Var[w_{t-1}], \end{aligned}$$

$$\begin{aligned} Cov[f_{t-1} - C, \pi_t] =& \bar{\pi}_1 Cov[A_t, f_{t-1}] + \bar{\pi}_2 Cov[A_{t-1}, f_{t-1}] \\ &+ \left(\bar{\pi}_3 + \bar{\pi}_{13}(E[A_t] - \bar{A}) + \bar{\pi}_{23}(E[A_{t-1}] - \bar{A})\right) Cov[w_t, f_{t-1}] \\ &+ \left(\bar{\pi}_4 + \bar{\pi}_{14}(E[A_t] - \bar{A}) + \bar{\pi}_{24}(E[A_{t-1}] - \bar{A})\right) Cov[w_{t-1}, f_{t-1}], \end{aligned}$$

and so on. Following that analysis, we obtain the regression coefficients:

$$\hat{\theta}_{w_t} = \bar{\pi}_3 + \bar{\pi}_{13}(E[A_t] - \bar{A}) + \bar{\pi}_{23}(E[A_{t-1}] - \bar{A}) + \omega \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \bar{\pi}_1,$$

$$\begin{aligned} \hat{\theta}_{w_{t-1}} =& \bar{\pi}_4 + \bar{\pi}_{14}(E[A_t] - \bar{A}) + \bar{\pi}_{24}(E[A_{t-1}] - \bar{A}) \\ &+ \omega \frac{\bar{\pi}_{13} + \beta\bar{\pi}_{24} + \beta\bar{\pi}_{14}\frac{\bar{\pi}_{12}}{-\bar{\pi}_{11} - \beta\bar{\pi}_{22} - \beta\bar{\pi}_{12}\lambda}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \left(1 - \beta\frac{\bar{\pi}_{12}}{\chi}\right) \bar{\pi}_2 + \omega \frac{\bar{\pi}_{14}}{-\bar{\pi}_{11} - (1+\beta)\bar{\pi}_{12} - \beta\bar{\pi}_{22}} \bar{\pi}_1. \end{aligned}$$

The other coefficients are unchanged. Under the assumption that at least one of $\bar{\pi}_{13}, \bar{\pi}_{14}, \bar{\pi}_{23}, \bar{\pi}_{24}$ is strictly positive, we have increased $\hat{\theta}_{w_t}$ and $\hat{\theta}_{w_{t-1}}$ if average actions are above $\bar{A}$ and have decreased $\hat{\theta}_{w_t}$ and $\hat{\theta}_{w_{t-1}}$ if average actions are below $\bar{A}$. The results follow.

I Proof of Proposition 6

We derive estimators in the case of $K = 0$, $K = 1$, and $K = 2$. The superscript on the estimators will indicate K.

Begin with $K = 0$. Demeaned to account for fixed effects, each row of the matrix $\tilde{X}$ of covariates is now simply $w_{jt} - C$, with the rows corresponding to the J observations. Therefore

$$E[\tilde{X}^T \tilde{X}] = J\zeta^2(\sigma^2 + \tau^2)$$

and

$$E[\tilde{X}^T \tilde{X}]^{-1} = \frac{1}{J\zeta^2(\sigma^2 + \tau^2)}.$$

We also have:

$$E[\tilde{X}^T \pi_{jt}] = J\, Cov[w_{jt} - C, \pi_{jt}].$$

We analyzed this covariance in the proof of Proposition 4 under the same assumptions. Using those results, we find that

$$\begin{aligned}\hat{\Phi}^0_{w_t} =& \bar{\pi}_3 + \bar{\pi}_4 \frac{\rho}{\sigma^2 + \tau^2} + \hat{\Gamma}_1 \left[\bar{\pi}_1 + \left(1 - \beta\frac{\bar{\pi}_{12}}{\chi}\right) \frac{\rho}{\sigma^2 + \tau^2}\bar{\pi}_2\right] \\ &+ \hat{\Gamma}_2 \bar{\pi}_1 \frac{\rho}{\sigma^2 + \tau^2} + \hat{\Gamma}_3 \left[\bar{\pi}_1 \frac{\rho}{\sigma^2 + \tau^2} + \left(1 - \beta\frac{\bar{\pi}_{12}}{\chi}\right) \frac{\tau^2}{\sigma^2 + \tau^2}\bar{\pi}_2\right].\end{aligned}$$

Now consider $K = 1$. Demeaned to account for fixed effects, each row of the matrix $\tilde{X}$ of covariates is

$$\begin{bmatrix} w_{jt} - C & w_{j(t-1)} - C \end{bmatrix},$$

with the rows corresponding to the J observations. Thus,

$$E[\tilde{X}^T \tilde{X}] = J\zeta^2 \begin{bmatrix} \sigma^2 + \tau^2 & \rho \\ \rho & \sigma^2 + \tau^2 \end{bmatrix}$$

and

$$E[\tilde{X}^T \tilde{X}]^{-1} = \frac{1}{J\zeta^2[(\sigma^2 + \tau^2)^2 - \rho^2]} \begin{bmatrix} \sigma^2 + \tau^2 & -\rho \\ -\rho & \sigma^2 + \tau^2 \end{bmatrix}.$$

We also have:

$$E[\tilde{X}^T \pi_{jt}] = J \begin{bmatrix} Cov[w_{jt} - C, \pi_{jt}] \\ Cov[w_{j(t-1)} - C, \pi_{jt}] \end{bmatrix}.$$

We analyzed these covariances in the proof of Proposition 4 under the same assumptions. Using those results, we find that

$$\begin{aligned}\hat{\Phi}^1_{w_t} =&\bar{\pi}_3 + \hat{\Gamma}_1\left\{\bar{\pi}_1 - \frac{\rho^2}{(\sigma^2+\tau^2)^2-\rho^2}\frac{\bar{\pi}_{12}}{\chi}\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2\right\} - \hat{\Gamma}_2\frac{\rho^2}{(\sigma^2+\tau^2)^2-\rho^2}\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2\\ &+\hat{\Gamma}_3\left\{\frac{\rho(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2}\bar{\pi}_1 + \left(\frac{\tau^2(\sigma^2+\tau^2)-\rho^2}{(\sigma^2+\tau^2)^2-\rho^2} - \frac{\rho\tau^2}{(\sigma^2+\tau^2)^2-\rho^2}\frac{\bar{\pi}_{12}}{\chi}\right)\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2\right\},\end{aligned}$$

$$\begin{aligned}\hat{\Phi}^1_{w_{t-1}} =&\bar{\pi}_4 + \hat{\Gamma}_1\left(1+\frac{\rho(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2}\frac{\bar{\pi}_{12}}{\chi}\right)\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2 + \hat{\Gamma}_2\left[\bar{\pi}_1 + \frac{\rho(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2}\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2\right]\\ &+\hat{\Gamma}_3\left\{\frac{-\rho^2}{(\sigma^2+\tau^2)^2-\rho^2}\bar{\pi}_1 + \left(\frac{\rho\sigma^2}{(\sigma^2+\tau^2)^2-\rho^2} + \frac{\tau^2(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2}\frac{\bar{\pi}_{12}}{\chi}\right)\left(1-\beta\frac{\bar{\pi}_{12}}{\chi}\right)\bar{\pi}_2\right\}.\end{aligned}$$

Finally, consider $K = 2$. Demeaned to account for fixed effects, each row of the matrix $\tilde{X}$ of covariates is

$$\begin{bmatrix} w_{jt} - C & w_{j(t-1)} - C & w_{j(t-2)} - C \end{bmatrix},$$

with the rows corresponding to the J observations. Thus,

$$E[\tilde{X}^T\tilde{X}] = J\zeta^2\begin{bmatrix}\sigma^2+\tau^2 & \rho & 0\\ \rho & \sigma^2+\tau^2 & \rho\\ 0 & \rho & \sigma^2+\tau^2\end{bmatrix}$$

and

$$E[\tilde{X}^T\tilde{X}]^{-1} = \frac{1}{J\zeta^2[(\sigma^2+\tau^2)^2-2\rho^2]}\begin{bmatrix}\sigma^2+\tau^2-\frac{\rho^2}{\sigma^2+\tau^2} & -\rho & \frac{\rho^2}{\sigma^2+\tau^2}\\ -\rho & \sigma^2+\tau^2 & -\rho\\ \frac{\rho^2}{\sigma^2+\tau^2} & -\rho & \sigma^2+\tau^2-\frac{\rho^2}{\sigma^2+\tau^2}\end{bmatrix}.$$

We also have:

$$E[\tilde{X}^T\pi_{jt}] = J\begin{bmatrix}Cov[w_{jt}-C,\pi_{jt}]\\ Cov[w_{j(t-1)}-C,\pi_{jt}]\\ Cov[w_{j(t-2)}-C,\pi_{jt}]\end{bmatrix}.$$

We analyzed these covariances in the proof of Proposition 4 under the same assumptions.

Using those results, we find that

$$\begin{aligned}\hat{\Phi}^2_{w_t} =&\bar{\pi}_3 + \hat{\Gamma}_1\left\{\bar{\pi}_1 + \frac{\rho}{\sigma^2+\tau^2}\frac{\rho^2}{(\sigma^2+\tau^2)^2-2\rho^2}\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\right\}\\
&+\hat{\Gamma}_3\frac{1}{(\sigma^2+\tau^2)^2-2\rho^2}\left\{\left(\sigma^2+\tau^2-\frac{\rho^2}{\sigma^2+\tau^2}\right)\left(\bar{\pi}_1\rho+\bar{\pi}_2\tau^2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\tau^2\right)\right.\\
&\qquad\left.+\left(\frac{\rho^2}{\sigma^2+\tau^2}\frac{\bar{\pi}_{12}}{\chi}-\rho\right)\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left(\rho+\frac{\bar{\pi}_{12}}{\chi}\tau^2\right)\right\}\\
&+\hat{\Gamma}_2\frac{\rho^2}{(\sigma^2+\tau^2)^2-2\rho^2}\frac{\rho}{\sigma^2+\tau^2}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi},\end{aligned}$$

$$\begin{aligned}\hat{\Phi}^2_{w_{t-1}} =&\bar{\pi}_4 + \hat{\Gamma}_1\left\{1-\frac{\rho^2}{(\sigma^2+\tau^2)^2-2\rho^2}\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\right\}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\\
&+\hat{\Gamma}_3\frac{1}{(\sigma^2+\tau^2)^2-2\rho^2}\left\{-\rho^2\bar{\pi}_1+\sigma^2\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\rho+[\tau^2(\sigma^2+\tau^2)-\rho^2]\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\right.\\
&\qquad\left.-\rho\tau^2\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\right\}\\
&+\hat{\Gamma}_2\left\{\bar{\pi}_1-\frac{\rho^2}{(\sigma^2+\tau^2)^2-2\rho^2}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}\right\},\end{aligned}$$

$$\begin{aligned}\hat{\Phi}^2_{w_{t-2}} =&\hat{\Gamma}_1\left\{\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\bar{\pi}_{12}}{\chi}+\frac{\rho\,(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-2\rho^2}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\right\}\\
&+\hat{\Gamma}_3\frac{1}{(\sigma^2+\tau^2)^2-2\rho^2}\\
&\quad\left\{\frac{\rho^2}{\sigma^2+\tau^2}\bar{\pi}_1\rho-\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\frac{\sigma^2\rho^2}{\sigma^2+\tau^2}+\rho\left(\sigma^2-\frac{\rho^2}{\sigma^2+\tau^2}\right)\frac{\bar{\pi}_{12}}{\chi}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\right.\\
&\qquad\left.+\tau^2\left(\sigma^2+\tau^2-\frac{\rho^2}{\sigma^2+\tau^2}\right)\left(\frac{\bar{\pi}_{12}}{\chi}\right)^2\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right)\right\}\\
&+\hat{\Gamma}_2\left\{1+\frac{(\sigma^2+\tau^2)^2-\rho^2}{(\sigma^2+\tau^2)^2-2\rho^2}\frac{\rho}{\sigma^2+\tau^2}\frac{\bar{\pi}_{12}}{\chi}\right\}\left(\bar{\pi}_2+\bar{\pi}_1\frac{\bar{\pi}_{12}}{\chi}\right).\end{aligned}$$

We now prove the parts of the proposition:

1. If Assumption 3 holds, then the Euler equation requires $\bar{\pi}_2 = \bar{\pi}_1 = 0$. The result follows by inspection and by recognizing that $\rho \leq \sigma\tau$ (because correlation coefficients are bounded above by 1).

2. If $\bar{\pi}_{12} = 0$, then

$$\hat{\Phi}^0_{w_t} = \bar{\pi}_3 + \bar{\pi}_4 \frac{\rho}{\sigma^2+\tau^2} + \hat{\Gamma}_1 \left[\bar{\pi}_1 + \frac{\rho}{\sigma^2+\tau^2} \bar{\pi}_2 \right] + \hat{\Gamma}_2 \bar{\pi}_1 \frac{\rho}{\sigma^2+\tau^2} + \hat{\Gamma}_3 \left[\bar{\pi}_1 \frac{\rho}{\sigma^2+\tau^2} + \frac{\tau^2}{\sigma^2+\tau^2} \bar{\pi}_2 \right],$$

$$\hat{\Phi}^1_{w_t} = \bar{\pi}_3 + \hat{\Gamma}_1 \bar{\pi}_1 - \hat{\Gamma}_2 \frac{\rho^2}{(\sigma^2+\tau^2)^2 - \rho^2} \bar{\pi}_2 + \hat{\Gamma}_3 \left\{ \frac{\rho(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2} \bar{\pi}_1 + \frac{\tau^2(\sigma^2+\tau^2)-\rho^2}{(\sigma^2+\tau^2)^2-\rho^2} \bar{\pi}_2 \right\},$$

$$\hat{\Phi}^1_{w_{t-1}} = \bar{\pi}_4 + \hat{\Gamma}_1 \bar{\pi}_2 + \hat{\Gamma}_2 \left[\bar{\pi}_1 + \frac{\rho(\sigma^2+\tau^2)}{(\sigma^2+\tau^2)^2-\rho^2} \bar{\pi}_2 \right] + \hat{\Gamma}_3 \left\{ \frac{-\rho^2}{(\sigma^2+\tau^2)^2-\rho^2} \bar{\pi}_1 + \frac{\rho\sigma^2}{(\sigma^2+\tau^2)^2-\rho^2} \bar{\pi}_2 \right\},$$

$$\hat{\Phi}^2_{w_t} = \bar{\pi}_3 + \hat{\Gamma}_1 \bar{\pi}_1 + \hat{\Gamma}_3 \frac{1}{(\sigma^2+\tau^2)^2 - 2\rho^2} \left\{ \frac{(\sigma^2+\tau^2)^2-\rho^2}{\sigma^2+\tau^2} \left(\bar{\pi}_1 \rho + \bar{\pi}_2 \tau^2 \right) - \rho^2 \bar{\pi}_2 \right\},$$

$$\hat{\Phi}^2_{w_{t-1}} = \bar{\pi}_4 + \hat{\Gamma}_1 \bar{\pi}_2 + \hat{\Gamma}_2 \bar{\pi}_1 + \hat{\Gamma}_3 \frac{\rho}{(\sigma^2+\tau^2)^2 - 2\rho^2} \left\{ -\rho \bar{\pi}_1 + \sigma^2 \bar{\pi}_2 \right\},$$

$$\hat{\Phi}^2_{w_{t-2}} = \hat{\Gamma}_2 \bar{\pi}_2 + \hat{\Gamma}_3 \frac{\rho}{\sigma^2+\tau^2} \frac{\rho}{(\sigma^2+\tau^2)^2 - 2\rho^2} \left\{ \bar{\pi}_1 \rho - \bar{\pi}_2 \sigma^2 \right\}.$$

By Proposition 2, $\bar{\pi}_{12} = 0$ implies $\mathrm{d}\bar{A}/\,\mathrm{d}C = \hat{\Gamma}_1 + \hat{\Gamma}_2 + \hat{\Gamma}_3$. If, in addition, $\beta\bar{\pi}_{23} = 0$, then $\hat{\Gamma}_3 = 0$. Result (a) follows from equation (4). $\bar{\pi}_{14} = 0$ implies $\hat{\Gamma}_2 = 0$. Result (b) follows from equation (4).

If $\bar{\pi}_{12} = 0$, $\beta\bar{\pi}_{23} = 0$, and $\bar{\pi}_4 = 0$, then

$$\hat{\Phi}^0_{w_t} = \bar{\pi}_3 + \hat{\Gamma}_1 \left[\bar{\pi}_1 + \frac{\rho}{\sigma^2+\tau^2} \bar{\pi}_2 \right]$$

and

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} = \bar{\pi}_3 + \hat{\Gamma}_1 \left[\bar{\pi}_1 + \bar{\pi}_2 \right].$$

We then have:

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} > \hat{\Phi}^0_{w_t} \quad \Leftrightarrow \quad \hat{\Gamma}_1 \bar{\pi}_2 > \hat{\Gamma}_1 \frac{\rho}{\sigma^2+\tau^2} \bar{\pi}_2.$$

If $\bar{\pi}_{13} > 0$, then $\hat{\Gamma}_1 > 0$ and

$$\frac{\mathrm{d}E_0[\pi_t]}{\mathrm{d}C} > \hat{\Phi}^0_{w_t} \quad \Leftrightarrow \quad \bar{\pi}_2 > \frac{\rho}{\sigma^2+\tau^2}\bar{\pi}_2.$$

Because correlation coefficients are bounded above by 1, $\rho \leq \sigma\tau$. Therefore

$$\bar{\pi}_2 > \frac{\rho}{\sigma^2+\tau^2}\bar{\pi}_2 \quad \Leftrightarrow \quad \bar{\pi}_2 > 0.$$

We have established result (c).

J Proof of Proposition 7

Observe that

$$\hat{\Lambda} = \frac{Cov[\tilde{\pi}_{jt}, \tilde{w}_{jt}]}{Var[\tilde{w}_{jt}]}.$$

If either Assumption 3 holds or $\bar{\pi}_{12}, \rho = 0$, then $Cov[\pi_t, w_{t-k}] = 0$ for $k > 2$. We then have:

$$\begin{aligned}
Cov[\tilde{\pi}_{jt}, \tilde{w}_{jt}] =& \frac{1}{\Delta^2}\Bigg\{ \sum_{T=t+2}^{t+\Delta-2} \Big(Cov[\pi_{jT}, w_{jT}] + Cov[\pi_{jT}, w_{j(T-1)}] + Cov[\pi_{jT}, w_{j(T-2)}] + Cov[\pi_{jT}, f_{jT}] \Big) \\
&+ Cov[\pi_{j(t+1)}, w_{j(t+1)}] + Cov[\pi_{j(t+1)}, w_{jt}] + Cov[\pi_{j(t+1)}, f_{j(t+1)}] \\
&+ Cov[\pi_{jt}, w_{jt}] + Cov[\pi_{jt}, f_{jt}] \\
&+ Cov[\pi_{j(t+\Delta-1)}, w_{j(t+\Delta-1)}] + Cov[\pi_{j(t+\Delta-1)}, w_{j(t+\Delta-2)}] + Cov[\pi_{j(t+\Delta-1)}, w_{j(t+\Delta-3)}] \Bigg\}
\end{aligned}$$

for $\Delta > 2$.

Begin by considering the case in which Assumption 3 holds. The Euler equation requires $\bar{\pi}_2 = \bar{\pi}_1 = 0$. Using intermediate results in the proof of Proposition 4, we have:

$$\frac{1}{\zeta^2} Cov[w_{jt}, \pi_{jt}] = (\sigma^2+\tau^2)\bar{\pi}_3 + \rho\bar{\pi}_4,$$

$$\frac{1}{\zeta^2} Cov[f_{jt}, \pi_{jt}] = \rho\bar{\pi}_3,$$

$$\frac{1}{\zeta^2} Cov[w_{j(t-1)}, \pi_{jt}] = \rho\bar{\pi}_3 + \bar{\pi}_4(\sigma^2+\tau^2),$$

$$\frac{1}{\zeta^2} Cov[w_{j(t-2)}, \pi_{jt}] = \rho\bar{\pi}_4.$$

We then have:

$$\begin{aligned}Cov[\tilde{\pi}_{jt}, \tilde{w}_{jt}] =&\frac{1}{\Delta^2}\left\{\Delta[\sigma^2+\tau^2+2\rho][\bar{\pi}_3+\bar{\pi}_4] - 2\rho\bar{\pi}_3 - [\sigma^2+\tau^2+2\rho]\bar{\pi}_4\right\}\\ =&\frac{1}{\Delta^2}\left\{[\sigma^2+\tau^2][\Delta\bar{\pi}_3+(\Delta-1)\bar{\pi}_4] + 2\rho[\Delta-1][\bar{\pi}_3+\bar{\pi}_4]\right\}.\end{aligned}$$

Note that

$$Var(\tilde{w}_{jt}) = \frac{\Delta(\sigma^2+\tau^2)+2\rho(\Delta-1)}{\Delta^2}.$$

The estimator is then:

$$\begin{aligned}\hat{\Lambda} =&\frac{[\sigma^2+\tau^2][\Delta\bar{\pi}_3+(\Delta-1)\bar{\pi}_4] + 2\rho[\Delta-1][\bar{\pi}_3+\bar{\pi}_4]}{\Delta(\sigma^2+\tau^2)+2\rho(\Delta-1)}\\ =&\bar{\pi}_3 + \bar{\pi}_4\frac{\Delta-1}{\Delta}\underbrace{\frac{\sigma^2+\tau^2+2\rho}{\sigma^2+\tau^2+2\rho\frac{\Delta-1}{\Delta}}}_{\triangleq\Upsilon_1}.\end{aligned}$$

As $\Delta \to \infty$,

$$\hat{\Lambda} \to \bar{\pi}_3 + \bar{\pi}_4,$$

which is also the marginal effect of climate when Assumption 3 holds.

Now consider the case in which Assumption 3 need not hold but $\bar{\pi}_{12} = 0$ and $\rho = 0$. Using intermediate results in the proof of Proposition 4, we have:

$$\frac{1}{\zeta^2}Cov[w_{jt}, \pi_{jt}] =(\sigma^2+\tau^2)\bar{\pi}_3 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}(\sigma^2+\tau^2)\bar{\pi}_1 + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_2\tau^2,$$

$$\frac{1}{\zeta^2}Cov[f_{jt}, \pi_{jt}] =\frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_1\tau^2,$$

$$\frac{1}{\zeta^2}Cov[w_{j(t-1)}, \pi_{jt}] =\bar{\pi}_4(\sigma^2+\tau^2) + \bar{\pi}_1\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}(\sigma^2+\tau^2) + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_2\left(\sigma^2+\tau^2\right),$$

$$\frac{1}{\zeta^2}Cov[w_{j(t-2)}, \pi_{jt}] =\bar{\pi}_2\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}(\sigma^2+\tau^2).$$

Summing these, we have:

$$\begin{aligned}&Cov[w_{jt}, \pi_{jt}] + Cov[f_{jt}, \pi_{jt}] + Cov[w_{j(t-1)}, \pi_{jt}] + Cov[w_{j(t-2)}, \pi_{jt}]\\ =&(\sigma^2+\tau^2)\left\{\bar{\pi}_3+\bar{\pi}_4+\frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}[\bar{\pi}_1+\bar{\pi}_2] + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}[\bar{\pi}_1+\bar{\pi}_2]\right\},\end{aligned}$$

$$\begin{aligned}&Cov[w_{j(t+1)}, \pi_{j(t+1)}] + Cov[f_{j(t+1)}, \pi_{j(t+1)}] + Cov[w_{jt}, \pi_{j(t+1)}]\\ =&(\sigma^2+\tau^2)\left\{\bar{\pi}_3 + \bar{\pi}_4 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}[\bar{\pi}_1+\bar{\pi}_2] + \bar{\pi}_1\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}} + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}[\bar{\pi}_1+\bar{\pi}_2]\right\},\end{aligned}$$

$$Cov[w_{jt}, \pi_{jt}] + Cov[f_{jt}, \pi_{jt}] =(\sigma^2+\tau^2)\left\{\bar{\pi}_3 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_1 + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}[\bar{\pi}_1+\bar{\pi}_2]\right\},$$

$$\begin{aligned}&Cov[w_{j(t+\Delta-1)}, \pi_{j(t+\Delta-1)}] + Cov[w_{j(t+\Delta-2)}, \pi_{j(t+\Delta-1)}] + Cov[w_{j(t+\Delta-3)}, \pi_{j(t+\Delta-1)}]\\ =&(\sigma^2+\tau^2)\left\{\bar{\pi}_3 + \bar{\pi}_4 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}[\bar{\pi}_1+\bar{\pi}_2] + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}\bar{\pi}_2\right\}.\end{aligned}$$

Note that

$$Var(\tilde{w}_{jt}) = \frac{\sigma^2+\tau^2}{\Delta}.$$

The estimator is then:

$$\begin{aligned}\hat{\Lambda} =&\bar{\pi}_3 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_1 + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}\bar{\pi}_2\\ &+\frac{\Delta-1}{\Delta}\left\{\bar{\pi}_4 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_2 + \bar{\pi}_1\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}} + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}\bar{\pi}_1\right\}\\ &+\frac{\Delta-2}{\Delta}\bar{\pi}_2\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\\ =&\hat{\Phi}^0_{w_t} + \frac{\Delta-1}{\Delta}\underbrace{\left\{\bar{\pi}_4 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\bar{\pi}_2 + \bar{\pi}_1\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}} + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}\bar{\pi}_1\right\}}_{\triangleq\Upsilon_1}\\ &+\frac{\Delta-2}{\Delta}\underbrace{\bar{\pi}_2\frac{\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}}_{\triangleq\Upsilon_2}.\end{aligned}$$

As $\Delta \to \infty$,

$$\begin{aligned}\hat{\Lambda} \to&\bar{\pi}_3 + \bar{\pi}_4 + \frac{\bar{\pi}_{13}+\beta\bar{\pi}_{24}+\bar{\pi}_{14}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}[\bar{\pi}_1+\bar{\pi}_2] + \frac{\beta\bar{\pi}_{23}}{-\bar{\pi}_{11}-\beta\bar{\pi}_{22}}\frac{\tau^2}{\sigma^2+\tau^2}[\bar{\pi}_1+\bar{\pi}_2]\\ =&\bar{\pi}_3 + \bar{\pi}_4 + [\bar{\pi}_1+\bar{\pi}_2]\left[\hat{\Gamma}_1 + \hat{\Gamma}_2 + \frac{\tau^2}{\sigma^2+\tau^2}\hat{\Gamma}_3\right].\end{aligned}$$

The results follow from inspection and previous results on the marginal effect of climate.